1.) Fig. 1 shows the (2D) free surface flow in an inclined channel (note the direction of gravity!) which contains a submerged cylinder of radius a. Far away from the cylinder the flow becomes uniform and has a parabolic velocity profile such that $\mathbf{u} = Uy(2H-y)/H^2\mathbf{e}_x$ as $x \to \pm\infty$. The undisturbed free surface height above the channel bed (at $y = 0$) is H and the centre of the cylinder is located at $x = 0$ and $y = b$. Gravity acts vertically downwards and surface tension σ acts along the free surface. The air pressure above the fluid is zero.

(i) Formulate the problem in dimensional terms: Write down the governing equation and all boundary conditions.

(ii) Non-dimensionalise all quantities and derive the non-dimensional version of the problem. Thus show that the problem is governed by only six non-dimensional parameters, namely the Reynolds number $Re = Ua/\nu$, the Capillary number $Ca = U\mu/\sigma$, the Grasshoff number $Gr = \rho g a^2/(\mu U)$ and the geometrical parameters a/b and H/a and α (or equivalent combinations of those).

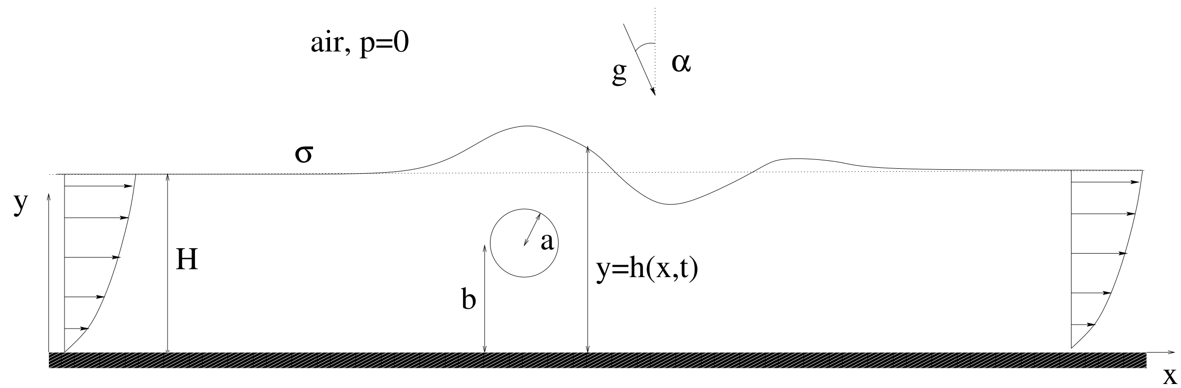

Figure 1: Free surface flow over a submerged cylinder.

2.) Viscous fluid of kinematic viscosity ν occupies the region $y > 0$ and is at rest for $t < 0$. For $t > 0$ a uniform constant tangential shear stress τ_0 is exerted on the fluid in the x-direction on the plane $y = 0$.

(i) Assuming that the shear stress induces a parallel flow of the form $\mathbf{u} = u(y,t)\mathbf{e}_x$ state the governing equations and the boundary and initial conditions for this problem [Be careful about the sign in the tangential shear stress condition].

(ii) Use linearity and dimensional arguments (in that order) to show that a similarity solution of the form $u = (\tau_0/\rho)\sqrt{t/\nu}f(\eta)$ must exist. The similarity variable η has to be determined as part of the solution. Show that the transformation to the similarity variable reduces the governing PDE to the ODE

$$2f'' + \eta f' - f = 0,$$

subject to the boundary conditions $f'(0) = -1$ and $f(\infty) = 0$. [Hint: The dimensions of the physical quantities are: $[\tau_0] = kg/(sec^2\ m)$, $[\rho] = kg/m^3$, $[\nu] = m^2/sec$, $[\mu] = kg/(sec\ m)$, $[y] = m$, $[t] = sec$].

(iii) A particular solution of the homogeneous ODE is given by $f_1(\eta) = \eta$. Use this to derive the second solution $f_2(\eta)$ and thus the general solution of the homogenous ODE.

(iv) [Not required for the coursework] Determine the two free constants from the boundary conditions.

Coursework

Please hand in the solutions to question 2 (i)-(iii) by 12 noon on Friday. Please place them into the envelope at the door of Dr. Heil's office (room 1.05 in the Lamb building).

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