MT3261: EXAMPLE SHEET[1] IV

1.) The interface between two media is positioned along the plane $x_3 = 0$. Medium (1) is a Newtonian incompressible fluid which occupies the region $x_3 \leq 0$.

(i) Assuming that the velocity and pressure fields in the fluid are given, determine the traction that medium (2) exerts onto the fluid along the interface.

(ii) Now assume that only the velocity field in the fluid is known and that medium (2) is an inviscid fluid under pressure $p^{(2)} = P$. What is the fluid pressure at the interface? [Remark: The (presumably) unexpected pressure difference is caused by the so-called 'viscous normal stress'].

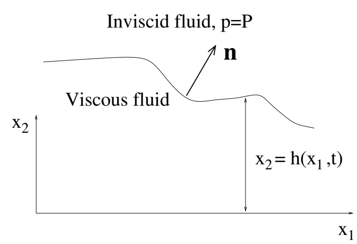
(iii) As in part (i), assume that the velocity and pressure fields in the fluid are known. However, we now replace the inviscid fluid in region (2) by an impermeable stationary solid body. What is the traction exerted onto the fluid at the interface in this case? [Hint: The fluid fulfills the no-slip and no-penetration conditions and the continuity equation along the *entire* interface.] Illustrate the situation with a 2D sketch which shows the direction of the applied traction and the corresponding velocity field near the interface.

2.) The figure below shows a 2D sketch of the interface between a viscous and an inviscid fluid.

(i) The curvature of a line specified in the form $y = h(x)$ is given by $\kappa = \pm h''/(1+(h')^2)^{3/2}$. Show that the minus sign has to be chosen if the dynamic boundary condition is to hold in the form

$$-Pn_i = \tau_{ij} n_j + \sigma \kappa n_i$$

where the normal vector is the outer unit normal vector on the fluid, as shown. [Hint: Restrict yourself to the hydrostatic case and consider the pressure jump induced by a curved interface for which $h'' > 0$].



(ii) Determine the two components of the outer unit normal $\mathbf{n}$ in terms of $h(x_1)$ and its derivatives.

(iii) We will now derive the dynamic boundary condition for a surface-tension-driven flow caused by a gently curved air-liquid interface for which $|h'|, |h''|, |\kappa| = \mathcal{O}(\epsilon) \ll 1$ and for zero external pressure, $P = 0$. The flow is driven by the surface-tension-induced pressure jump over the curved interface. Therefore, we assume that the resulting velocity gradients and the pressure are also small, i.e. $|\partial u_i/\partial x_j|, |p| = \mathcal{O}(\epsilon) \ll 1$.
Use these assumptions to determine the leading order terms for the components of the normal vector and the curvature κ and show that the leading order terms in the dynamic boundary condition are

$$\sigma \frac{\partial^2 h}{\partial x_1^2} + p = 2\mu \frac{\partial u_2}{\partial x_2} \quad \text{and} \quad \mu \left(\frac{\partial u_1}{\partial x_2} + \frac{\partial u_2}{\partial x_1} \right) = 0.$$

[Hint: Neglect any terms whose size is quadratic in ϵ against terms which are linear in this quantity, etc.].

Coursework

Please hand in the solution to question 1 by 12 noon on Friday. Please place them into the envelope at the door of Dr. Heil's office (room 1.05 in the Lamb building).

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