Mechanical oscillator with weak damping

- Governing (linear) ODE:

\[ \ddot{x} + \epsilon \dot{x} + x = 0 \]

subject to the initial conditions

\[ x(t = 0) = 1 \quad \text{and} \quad \dot{x}(t = 0) = 0. \]

Comparison between perturbation solution and exact solution for \( \epsilon = 0.2 \)

- One-term perturbation solution (red), exact solution (green):
Comparison between perturbation solution and exact solution for $\epsilon = 0.2$

- Two-term perturbation solution (red), exact solution (green):

- Three-term perturbation solution (red), exact solution (green):
Comparison between perturbation solution and exact solution for $\epsilon = 0.2$

- Four-term perturbation solution (red), exact solution (green):

- Agreement over a finite time-interval is very pleasing. However, over sufficiently large times, the perturbation solution diverges: