[Numerical] experiment: Finite-amplitude oscillation of an undamped pendulum

- Governing (non-linear!) ODE:
  \[ \ddot{\theta} + \sin \theta = 0 \]

subject to the initial conditions

\[ \theta(t = 0) = \epsilon \quad \text{and} \quad \dot{\theta}(t = 0) = 0. \]

- Plot for \( \epsilon = 0.1, 0.7, 1.4, 2.1 \):

- **Observation:** Period of the oscillation increases for larger amplitudes.
Comparison between perturbation solution and “exact” solution for $\epsilon = 1.2$

- One-term perturbation solution (red), exact solution (green):

- Two-term perturbation solution (red), exact solution (green):
Comparison between perturbation solution and “exact” solution for $\epsilon = 1.2$ (cont.)

- Three-term perturbation solution (red), exact solution (green):

- Four-term perturbation solution (red), exact solution (green):
Comparison between perturbation solution and “exact” solution for $\epsilon = 1.2$ (cont.)

- Four-term perturbation solution (red), exact solution (green):

- Agreement over a finite time-interval is very pleasing. However, over sufficiently large times, the perturbation solution diverges: