

# MATH10222: EXAMPLE SHEET<sup>1</sup> I

*Questions for supervision classes*

Hand in the solutions to *all* questions on this sheet for your supervision classes. If you have a new supervisor and the procedure for handing in solutions has not yet been established, attempt all questions and raise any problems that you encounter with your supervisor.

## 1. Classifying ODEs

Classify the following ODEs: Are they linear or nonlinear, autonomous or non-autonomous, and what is their order?

- (a)  $u'(x) + u^2(x) = \cos x$ .
- (b)  $\frac{d^4u(t)}{dt^4} + (2 + \cos(t)) u(t) = 3$ .
- (c)  $\ddot{\theta} + \Omega^2 \sin \theta = 0$ , where the constant  $\Omega$  is given.

## 2. Properties of ODEs

True or false? Briefly justify every answer. (Be careful – some of these are tricky!)

- (a)  $\left(\frac{d\phi}{ds}\right)^2 = 2s\phi$  is a second order ODE.
- (b)  $u''(x) + u'(x - 1) = 1$  is a second-order ODE.
- (c)  $\frac{dx}{dy} + 5xy^2 = 0$  is not a linear ODE.
- (d) The ODE  $y'^2 + y^2 = 0$  has no real solutions.
- (e)  $t^2 \frac{d^2t}{dz^2} + 2t \frac{dt}{dz} + 2t = 0$  is not an autonomous ODE.

## 3. Solutions of ODEs; Boundary and Initial Value Problems

Confirm that  $y = A_1 e^x + A_2(1 + x)$  is a solution of the ODE  $xy'' - (1 + x)y' + y = 0$ .

Find the values of the constants  $A_1$  and  $A_2$  that are determined by each of the following pairs of constraints. For each case, state whether the combination of the ODE and the constraint constitutes a boundary value problem or an initial value problem.

- (a)  $y(0) = 1$  and  $y(1) = 1$ .
- (b)  $y(1) = 0$  and  $y'(2) = 0$ .
- (c)  $y'(1) = e$  and  $y'(1) = y(1)$ .

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