MATH10222: EXAMPLE SHEET[1] 0

Questions for supervision classes

This example sheet contains a few "warm up" exercises for the course. None of the questions are technically difficult – if you think they are, you should start some serious revision **now**!
Question 2 introduces the main topic for the first half of MATH10222. With your current knowledge, solving the question may require a bit of trial and error. During the first six weeks of the course we shall develop methods that allow you to answer questions of this type in a less *ad hoc* way.
Note that from next week, you are expected to hand in solutions to selected questions on the example sheets before the supervision. Make sure you establish the procedure for this with your supervisor.

1. **Integration as the inversion of differentiation**

   (a) Given the function
   $$y_1(x) = \sin(x),$$
   determine its derivative, dy_1/dx. Does this question have a unique answer?

   (b) Given that the derivative of the function $y_2(x)$ is
   $$\frac{dy_2}{dx} = \cos(x),$$
   determine the function $y_2(x)$ itself. Does this question have a unique answer? If not, how do the various solutions differ?

   (c) Given that the second derivative of the function $y_3(x)$ is given by
   $$\frac{d^2y_3}{dx^2} = -\sin(x),$$
   determine the function $y_3(x)$ itself. Does this question have a unique answer? If not, how do the various solutions differ?

2. **Ordinary differential equations (ODEs) as relations between a function and its derivatives**

   (a) Determine the function $y_4(x)$ that satisfies
   $$\frac{dy_4}{dx} = y_4(x).$$
   Another way to pose this question is to ask: "Which function is equal to its first derivative?". Is the answer to this question unique? If not, how do the various solutions differ?

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(b) Determine the function $y_5(x)$ that satisfies

$$\frac{d^2y_5}{dx^2} = -y_5(x).$$

Another way to pose this question is to ask: "Which function is equal to the negative of its second derivative?". Is the answer to this question unique? If not, how do the various solutions differ?