MATH10222/11222 Exam Feedback: ODEs

May 2015

A1 This question was generally well done.

- **(i)** A surprising number of people claimed the ODE was non-linear - perhaps they had been learning past exams rather than the actual material. As a result, many said that E&U was only guaranteed in the vicinity of the initial conditions, or over some interval (although if you then said that interval was the real numbers you redeemed yourself).
- **(ii)** Some struggled, but many seemed to use the hint and give answers of a high standard. Some forgot to look at isoclines in the left hand quadrants which led to incorrect integral curves. Others drew perfect isoclines but either didn't draw any integral curves or joined them incorrectly, most commonly adding an extra family of curves to the left and right of the actual ones. Note: even those unable to generate sketches could still earn marks if they were able to do calculations similar to the hint.
- **(iii)** Very few errors, although a surprising number of people couldn't integrate x. A few dealt with the constant of integration incorrectly when undoing the logarithm.

A2 This question was generally answered to a high standard.

- Quite a few people used the substitution $z = y/x$ despite not being able to write the ODE in the form $\frac{dy}{dx} = f(y/x)$. This then led to either using the integrating factor method anyway (you should have used it from the start) or having to make ANOTHER substitution ($w = z/x$). You should have been able to return the correct solution either way, but it was much more work than necessary.
- Most issues came from the right hand side of the equation, either forgetting to divide it by x when putting the ODE in the relevant form for the integrating factor method or forgetting to multiply it by the integrating factor when the time came.
- Some absorbed the $1/x^2$ factor of the second term into the constant of integration. Please don't.

A3 This question was generally answered strongly.

(i)
- Despite finding the roots to the characteristic equation $(-1 \pm \imath)$, some still gave the wrong homogeneous solution - usually putting the wrong argument in the exponential function. A few people attempted to find the constants in the homogeneous solution before they had found a particular solution, or made a mistake obtaining $\dot{y}$ (not using product rule).
- Many stuck to using cos and sin functions in the particular solution rather than using the hint, which was fine assuming you didn't mess up the algebra. A few forgot to take the imaginary part at the end (or took it incorrectly) using the complex variables method.
- Regarding the behaviour as $t \to \infty$, many ignored/overlooked this part of the question.

(ii)
- The vast majority did very well to spot the correct particular solution, although some included a constant or a t^4 term.
- A surprising number of people seemed to forget the form of the ODE when finding the particular solution, forgetting the 2 in front of the $\dot{y}$ term.
- Quite a few didn't just use the characteristic polynomial method again and instead substituted $v = \dot{y}$ and used the integrating factor method. While valid, this would lead to having to integrate a $t^2 e^t$ term which caused errors for many trying this.

A4 This was the question people struggled with the most in general, but was still answered to a fair standard by many. Some only managed to rearrange the ODE without solving at the different orders.

- Many failed to substitute the expansion into the initial conditions, which often led to people incorrectly assuming $x_1(0) = 1$ when dealing with the $O(\varepsilon)$ system.
- The most common error was from the inhomogeneous term in the $O(\epsilon)$ ODE, $(\dot{x}_0)^2$. Some forgot the derivative, others made a sign error.
- So many people still considered the ϵ^2 terms (many erroneously tried to use these terms to get x_1) despite the question telling you you only needed to consider the first two terms.

A5 This question was generally well answered.

- The vast majority of people spotted the correct substitution $(v = y')$, however some people then mixed up their notation for derivatives, using primes to represent both x and y derivatives, or just made errors in the form of v', usually stating it was $\frac{dv}{dy}$.

- Lots of people had trouble using the boundary conditions to get the constants of integration. Most correctly spotted the one from the second integration was zero, but many were flummoxed by the other constant appearing twice when the $y(1)$ condition was used. It seemed those who didn't make y the subject of the solution immediately struggled the most.