Mechanical oscillator with weak damping

• Governing (linear) ODE:

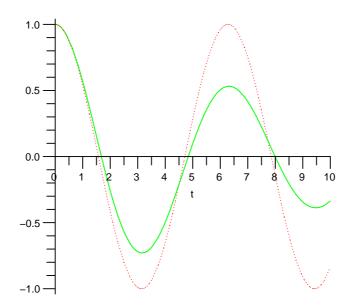
$$\ddot{x} + \epsilon \dot{x} + x = 0$$

subject to the initial conditions

$$x(t = 0) = 1$$
 and $\dot{x}(t = 0) = 0$.

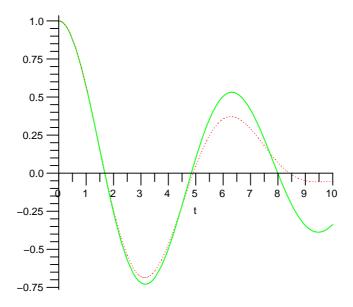
Comparison between perturbation solution and exact solution for $\epsilon = 0.2$

• One-term perturbation solution (red), exact solution (green):

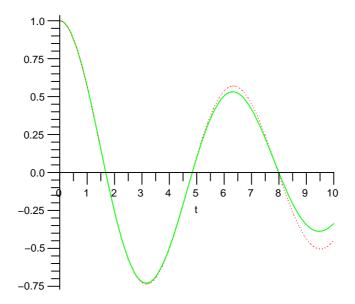


Comparison between perturbation solution and exact solution for $\epsilon = 0.2$

• Two-term perturbation solution (red), exact solution (green):

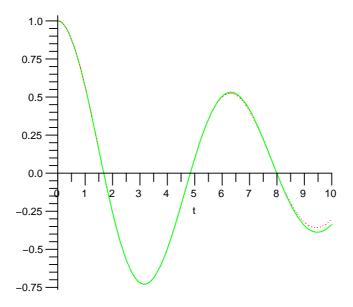


• Three-term perturbation solution (red), exact solution (green):



Comparison between perturbation solution and exact solution for $\epsilon = 0.2$

• Four-term perturbation solution (red), exact solution (green):



• Agreement over a finite time-interval is very pleasing. However, over sufficiently large times, the perturbation solution diverges:

