[Numerical] experiment: Finite-amplitude oscillation of an undamped pendulum

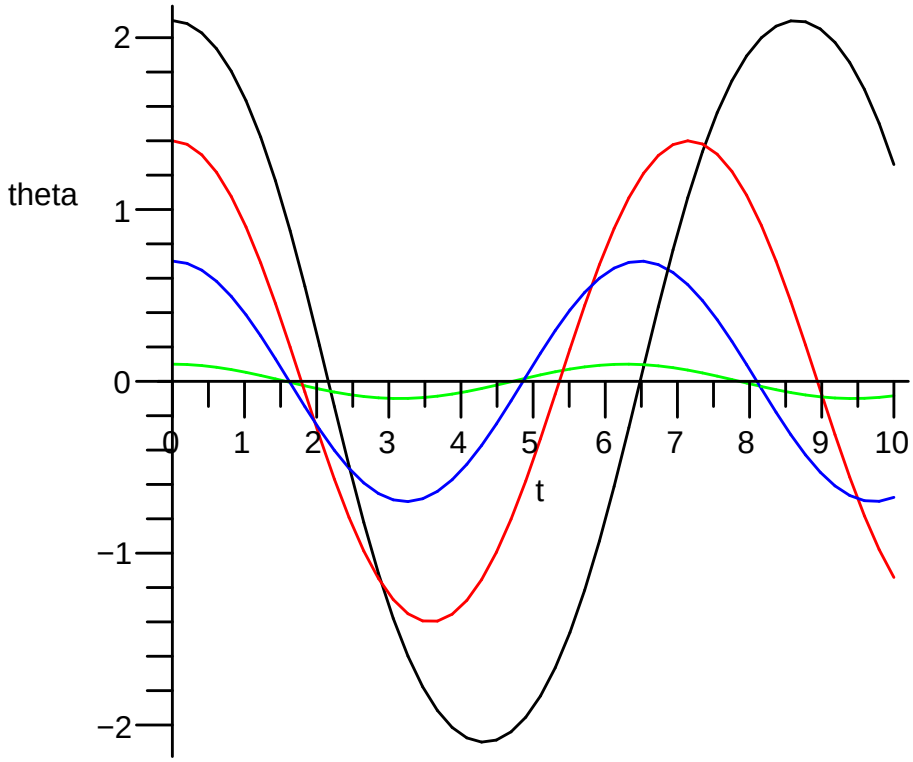
- Governing (non-linear!) ODE:

$$\ddot{\theta} + \sin\theta = 0$$

  subject to the initial conditions

$$\theta(t=0) = \epsilon \quad \text{and} \quad \dot{\theta}(t=0) = 0.$$

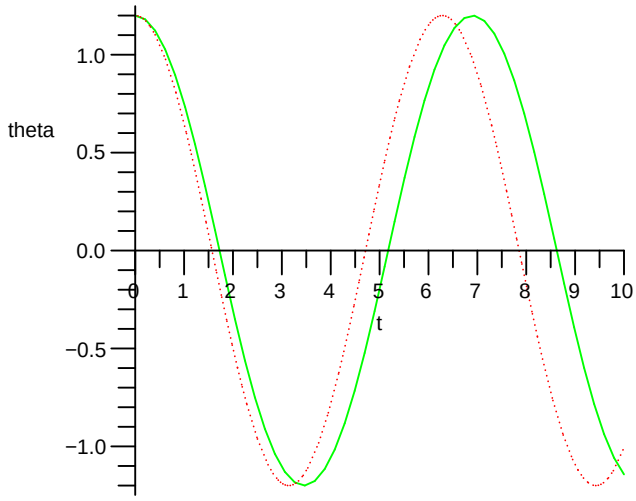
- Plot for $\epsilon = 0.1, 0.7, 1.4, 2.1$:



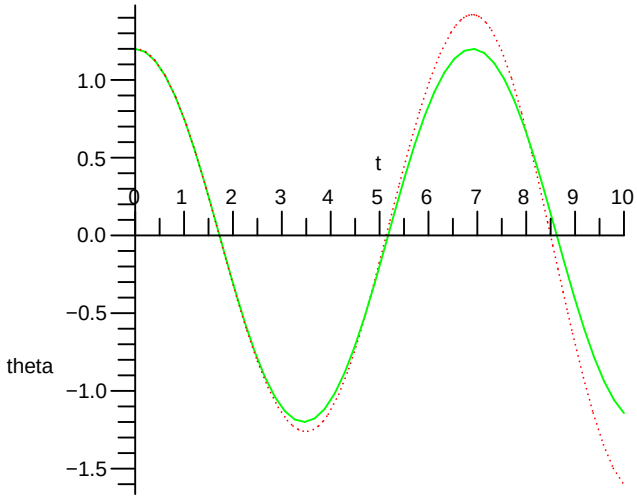
- **Observation:** Period of the oscillation increases for larger amplitudes.

Comparison between perturbation solution and "exact" solution for $\epsilon = 1.2$

- One-term perturbation solution (red), exact solution (green):

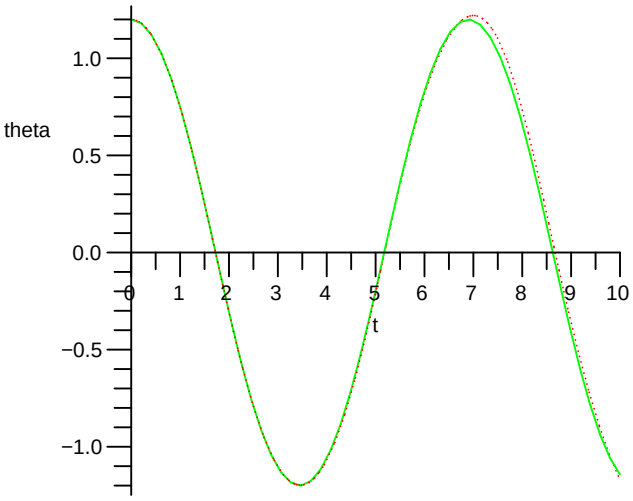


- Two-term perturbation solution (red), exact solution (green):

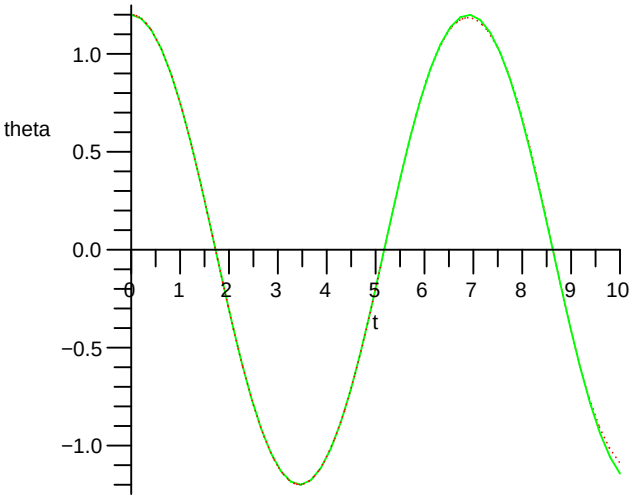


Comparison between perturbation solution and "exact" solution for $\epsilon = 1.2$ (cont.)

- Three-term perturbation solution (red), exact solution (green):

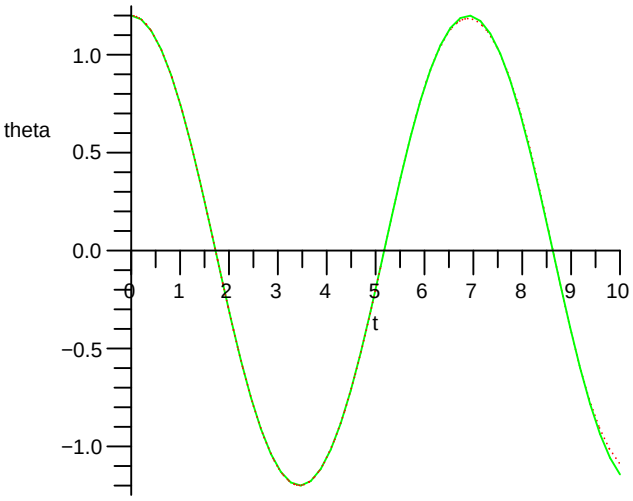


- Four-term perturbation solution (red), exact solution (green):



Comparison between perturbation solution and "exact" solution for $\epsilon = 1.2$ (cont.)

- Four-term perturbation solution (red), exact solution (green):



- Agreement over a finite time-interval is very pleasing. However, over sufficiently large times, the perturbation solution diverges: