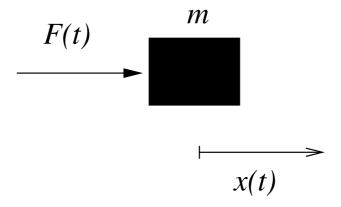
# Everything you always wanted to know about mechanical oscillators but were afraid to ask

- The first half of MATH10222 is not directly concerned with mechanics.
- However, mechanical systems provide nice illustrations of many of the phenomena that we have discussed (or will discuss) in a more abstract mathematical setting.

### I. Newton's law for one-dimensional motion

• In words: "The sum of all forces acting on a particle of mass m is equal to its mass times its acceleration"

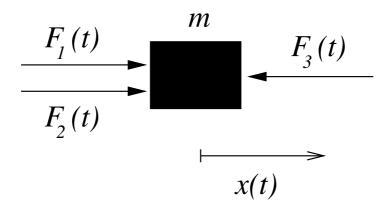


• Or, written as an equation:

$$m\frac{d^2x}{dt^2} = F(t)$$

## I. Newton's law for one-dimensional motion (cont.)

• Here's an example with multiple forces



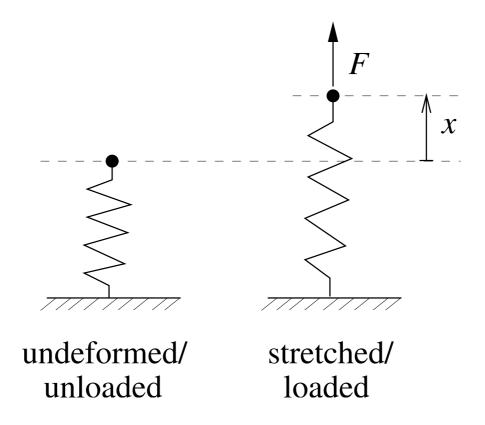
• In this case Newton's law becomes:

$$m\frac{d^2x}{dt^2} = F_1(t) + F_2(t) - F_3(t)$$

• Note the direction of the forces!

#### II. (Linearly) elastic springs

• Observation: When a spring is loaded by a force, F, its length increases by a certain amount, x, say.



• For a linearly elastic spring we have

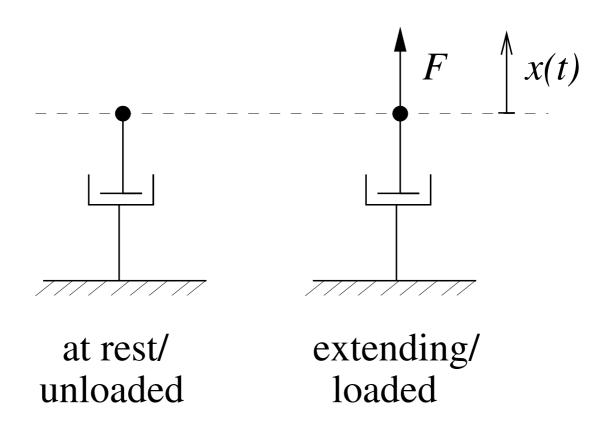
$$F = c x$$

where c is the "spring constant", a measure of its stiffness.

ullet Thus c indicates how strongly the spring resists its static extension.

#### III. (Linear) dampers

• Observation: When a damper is loaded by a force F its length increases at a rate dx/dt:



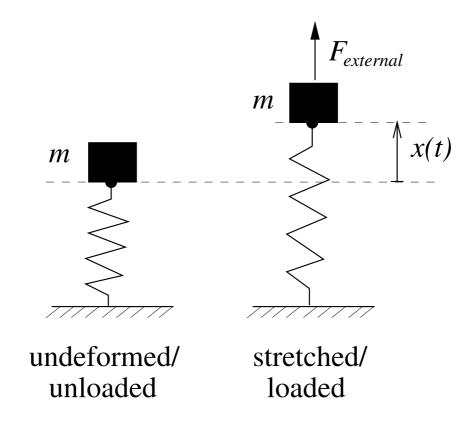
• For a linear damper we have

$$F = k \frac{dx}{dt}$$

where k is the "damping constant", a measure of how strongly the damper resists its dynamic extension.

### IV. Putting it all together: "Action = Reaction"

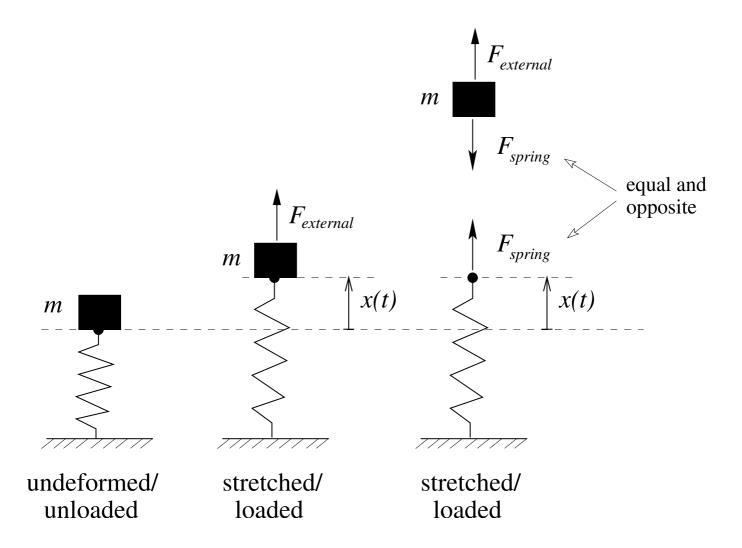
• Here is a mass m, attached to a spring of stiffness c, and loaded by a force,  $F_{external}$ .



- What is the equation of motion for the mass?
- Write down Newton's law for the mass.
- $\bullet \Longrightarrow$ What forces act on the mass?

#### IV. Putting it all together (cont.)

• "Action = Reaction": The spring pulls the mass and mass pulls the spring (in the opposite direction, obviously!):



• Thus Newton's law states

$$m\frac{d^2x}{dt^2} = F_{external} - F_{spring},$$

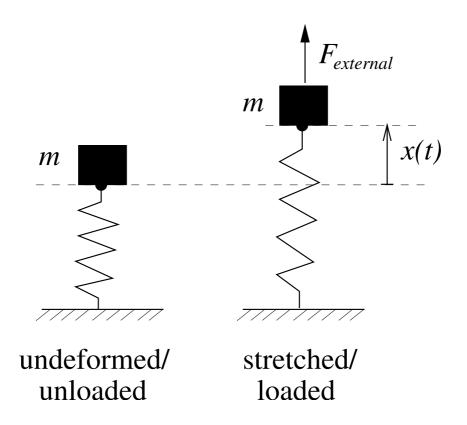
or, using what we've just learned about linear springs:

$$m\frac{d^2x}{dt^2} = F_{external} - cx.$$

#### IV. Putting it all together (cont.)

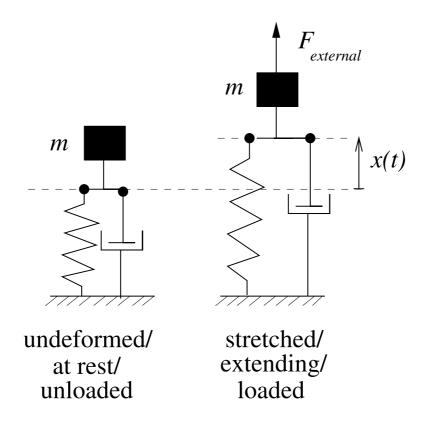
• Rewrite to the standard form of a second-order ODE for x(t):

$$m\frac{d^2x}{dt^2} + cx = F_{external}.$$



#### Exercise: Try it for yourself

• Here is a mass m, attached to a spring of stiffness c, and a damper (damping constant k), loaded by a force  $F_{external}$ .



• Show that the equation of motion for the mass is

$$m\frac{d^2x}{dt^2} + k\frac{dx}{dt} + cx = F_{external}$$