## [Numerical] experiment: Finite-amplitude oscillation of an undamped pendulum

- Governing (non-linear!) ODE:

$$
\ddot{\theta}+\sin \theta=0
$$

subject to the initial conditions

$$
\theta(t=0)=\epsilon \quad \text { and } \quad \dot{\theta}(t=0)=0
$$

- Plot for $\epsilon=0.1,0.7,1.4,2.1$ :

- Observation: Period of the oscillation increases for larger amplitudes.


## Comparison between perturbation solution and "exact" solution for $\epsilon=1.2$

- One-term perturbation solution (red), exact solution (green):

- Two-term perturbation solution (red), exact solution (green):



## Comparison between perturbation solution and "exact" solution for $\epsilon=1.2$ (cont.)

- Three-term perturbation solution (red), exact solution (green):

- Four-term perturbation solution (red), exact solution (green):



## Comparison between perturbation solution and "exact" solution for $\epsilon=1.2$ (cont.)

- Four-term perturbation solution (red), exact solution (green):

- Agreement over a finite time-interval is very pleasing. However, over sufficiently large times, the perturbation solution diverges:


