## [Numerical] experiment: Finite-amplitude oscillation of an undamped pendulum

• Governing (non-linear!) ODE:

$$\ddot{\theta} + \sin \theta = 0$$

subject to the initial conditions

$$\theta(t=0) = \epsilon$$
 and  $\theta(t=0) = 0$ .

• Plot for  $\epsilon = 0.1, 0.7, 1.4, 2.1$ :



• **Observation:** Period of the oscillation increases for larger amplitudes.

## Comparison between perturbation solution and "exact" solution for $\epsilon = 1.2$

• One-term perturbation solution (red), exact solution (green):



• Two-term perturbation solution (red), exact solution (green):



## Comparison between perturbation solution and "exact" solution for $\epsilon = 1.2$ (cont.)

• Three-term perturbation solution (red), exact solution (green):



• Four-term perturbation solution (red), exact solution (green):



## Comparison between perturbation solution and "exact" solution for $\epsilon = 1.2$ (cont.)

• Four-term perturbation solution (red), exact solution (green):



• Agreement over a finite time-interval is very pleasing. However, over sufficiently large times, the perturbation solution diverges:

