MATH10222: EXAMPLE SHEET[1] I

> *Questions for supervision classes*
>
> Hand in the solutions to *all* questions on this sheet for your supervision classes. If you have a new supervisor and the procedure for handing in solutions has not yet been established, attempt all questions and raise any problems that you encounter with your supervisor.

1. **Classifying ODEs**
   Classify the following ODEs: Are they linear or nonlinear, autonomous or non-autonomous, and what is their order?

   (a) $u'(x) + u^2(x) = \cos x$.

   (b) $\frac{\mathrm{d}^4 u(t)}{\mathrm{d}t^4} + (2 + \cos(t))\, u(t) = 3$.

   (c) $\ddot{\theta} + \Omega^2 \sin\theta = 0$, where the constant Ω is given.

2. **Properties of ODEs**
   True or false? Briefly justify every answer. (Be careful – some of these are tricky!)

   (a) $\left(\frac{\mathrm{d}\phi}{\mathrm{d}s}\right)^2 = 2s\phi$ is a second order ODE.

   (b) $u''(x) + u'(x-1) = 1$ is a second-order ODE.

   (c) $\frac{\mathrm{d}x}{\mathrm{d}y} + 5xy^2 = 0$ is not a linear ODE.

   (d) The ODE $y'^2 + y^2 = 0$ has no real solutions.

   (e) $t^2 \frac{\mathrm{d}^2 t}{\mathrm{d}z^2} + 2t\frac{\mathrm{d}t}{\mathrm{d}z} + 2t = 0$ is a not an autonomous ODE.

3. **Solutions of ODEs; Boundary and Initial Value Problems**
   Confirm that $y = A_1\, e^x + A_2(1+x)$ is a solution of the ODE $xy'' - (1+x)y' + y = 0$.

   Find the values of the constants A_1 and A_2 that are determined by each of the following pairs of constraints. For each case, state whether the combination of the ODE and the constraint constitutes a boundary value problem or an initial value problem.

   (a) $y(0) = 1$ and $y(1) = 1$.

   (b) $y(1) = 0$ and $y'(2) = 0$.

   (c) $y'(1) = e$ and $y'(1) = y(1)$.

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