Boundary and initial conditions

> An m-th order ODE must be augmented by m constraints (in the form of "boundary" or "initial" conditions) if there is to be a unique solution.

Notes:

- This is a necessary, not a sufficient condition: Even if an m-th order ODE is augmented by m constraints, there may be multiple (or no!) solutions.

Initial conditions (ICs)

- If all constraints are applied at the same value of the independent variable, we refer to them as *initial conditions.*

Boundary conditions (BCs)

- If the constraints are applied at multiple values of the independent variable (typically at the ends of the interval I in which the solution is sought), we refer to them as *boundary conditions.*

Boundary and initial value problems

Initial value problems (IVPs)

$$\text{``IVP} = \text{ODE} + \text{ICs''}$$

- Initial value problems (IVPs) typically describe evolution processes in which the initial state (at time $t = t_0$, say) of a system is characterised by the initial conditions, while the ODE describes the dynamics of its subsequent evolution.

Boundary value problems (BVPs)

$$\text{``BVP} = \text{ODE} + \text{BCs''}$$

- Boundary value problems (BVPs) typically describe spatial problems in which the boundary conditions describe the state of the system on the domain boundaries, while the ODE governs its behaviour in the interior of the domain.