

## MT3271: EXAMPLE SHEET<sup>1</sup> IX

- 1.) For a body in plane strain (parallel to  $z = 0$ ) let  $C_{AB}$  be a part of its boundary curve in the  $xy$ -plane, as shown in Fig. 1. Show that the resultant force  $(F_x, F_y)$  and the moment about the origin  $\mathbf{M}_0$

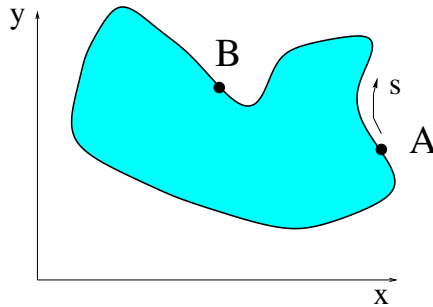


Figure 1: A rigid body in plane strain.

(all per unit length in the  $z$ -direction) of the tractions acting upon  $C_{AB}$  are given by

$$F_x = \left[ \frac{\partial \phi}{\partial y} \right]_A^B, \quad F_y = \left[ -\frac{\partial \phi}{\partial x} \right]_A^B$$

and

$$\mathbf{M}_0 = \left[ \phi - x \frac{\partial \phi}{\partial x} - y \frac{\partial \phi}{\partial y} \right]_A^B \mathbf{e}_z.$$

[Hint: Use integration by parts when determining  $\mathbf{M}_0$ ].

- 2.) a) Show that

$$\nabla^2(\phi\psi) = \phi\nabla^2\psi + \psi\nabla^2\phi + 2\nabla\psi \cdot \nabla\phi.$$

- b) Using the result from (a), show that the functions

$$F_1(x, y) = xH(x, y)$$

$$F_2(x, y) = yH(x, y)$$

$$F_3(x, y) = (x^2 + y^2)H(x, y)$$

are biharmonic if  $H(x, y)$  is a harmonic function, i.e.  $\nabla^2 H(x, y) = 0$ .

- 3.) Show that the general solution to the axisymmetric biharmonic equation

$$\tilde{\nabla}^4 \phi(r) = \phi_{,rrrr} + \frac{2}{r} \phi_{,rrr} - \frac{1}{r^2} \phi_{,rr} + \frac{1}{r^3} \phi_{,r} \quad (1)$$

is given by

$$\phi(r) = A_0 + B_0 r^2 + C_0 \ln r + D_0 r^2 \ln r.$$

[Hint: (1) is a linear Euler equation].

### Coursework

Please hand in the solution to question 1 by Friday. Please place them into the file in Dr. Heil's pigeonhole in the general office on the 4th floor.

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