

MT3271: EXAMPLE SHEET¹ V

- 1.) Show that in a homogeneous, isotropic, linearly elastic body the principal axes of the stress and strain tensors coincide. Derive the relation between principal strains and principal stresses.
- 2.) It was shown in the lecture that the stresses/tractions t_i acting at a certain point in the body, in a plane characterised by its unit normal vector n_j , are given by $t_i = \tau_{ij}n_j$. For a homogeneous, isotropic, linearly elastic solid, express the stresses/tractions in terms of the displacements u_i and thus show that

$$\mathbf{t} = \lambda \mathbf{n} \operatorname{div} \mathbf{u} + 2\mu(\mathbf{n} \cdot \nabla)\mathbf{u} + \mu\mathbf{n} \times \operatorname{curl} \mathbf{u}.$$

[Hints: (i) At some point you might find it useful to add and subtract $\mu \partial u_i / \partial x_j n_j$;
(ii) Recall that $\omega_{ij}n_j$ can be written in symbolic form as $\boldsymbol{\omega} \times \mathbf{n}$ where $\boldsymbol{\omega} = 1/2 \operatorname{curl} \mathbf{u}$.]

- 3.) Starting from the stress-strain relationships for a homogeneous, isotropic, linearly elastic body, $\tau_{ij} = \lambda \delta_{ij} e_{kk} + 2\mu e_{ij}$, derive the constitutive equation in terms of the two ‘engineering constants’ E and ν and thus show that:

$$\tau_{ij} = \frac{E}{1 + \nu} \left(e_{ij} + \frac{\nu}{1 - 2\nu} \delta_{ij} \underbrace{e_{kk}}_d \right)$$

and

$$e_{ij} = \frac{1}{E} \left((1 + \nu)\tau_{ij} - \nu \delta_{ij} \underbrace{\tau_{kk}}_\theta \right).$$

Coursework

Please hand in the solution to questions 1 and 2 by Friday. Please place them into the file in Dr. Heil’s pigeon-hole in the general office on the 4th floor.

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