MT3271: EXAMPLE SHEET[1] I

1.) Which one of these equations in index notation are valid? Remember the summation convention!

a) $c = a_i b_i$

b) $c = a_{ij} b_i$

c) $c_i = a_{ij} b_i$

d) $c_i = a_{ij} b_j$

e) $c_i = a_{ji} b_j$

f) $\sigma_{ij} = \alpha_{ij} T + E_{ijkl} e_{kl}$

g) $\sigma_{ij} = \alpha_{kl} T_i + E_{ijkl} e_{ij}$

h) $k_{ijkl} = a_i b_{kl} c_{njm} d_{mn} + e_{ik} e_{jn} f_{nl}$

2.) Using a comma to denote partial differentiation (e.g. $\partial u/\partial x_2 = u_{,2}$), transform the following expressions into index notation:

a) $\nabla u(x_1, x_2, x_3)$

b) $\underline{\mathbf{A}} = \nabla \mathbf{u}(x_1, x_2, x_3)$

c) $\nabla \cdot \mathbf{u}(x_1, x_2, x_3) = f(x_1, x_2, x_3)$

d) $\nabla^2 u(x_1, x_2, x_3) = f(x_1, x_2, x_3)$

e) $\nabla^2 \mathbf{u}(x_1, x_2, x_3) = \mathbf{f}(x_1, x_2, x_3)$

3.) A 2D body occupying the region $\{d : 0 \le x_1 \le 1, 0 \le x_2 \le 1\}$ is displaced by the following displacement fields:

a) $u_1 = \epsilon(x_1 + 2x_2);\ u_2 = \epsilon(3 + x_2)$ where $\epsilon \ll 1$

b) $u_1 = U_1 - x_1 + \sqrt{x_1^2 + x_2^2}\ \cos\left(\arctan\left(\frac{x_2}{x_1}\right) + \Phi\right);$
$u_2 = U_2 - x_2 + \sqrt{x_1^2 + x_2^2}\sin\left(\arctan\left(\frac{x_2}{x_1}\right) + \Phi\right)$ where U_1, U_2, Φ are constants.

Sketch the deformed body D (setting $\epsilon = 1$ in (a) for simplicity). For the displacement field (a) determine the displacement gradient tensor and the strain and rotation tensors. For the displacement field (b) determine the linear strain and rotation tensors for $\Phi \ll 1$ – Think before you calculate! [Hints: Does anything here smell of cylindrical polars? First consider the two special cases $U_1 = U_2 = 0$ and $\Phi = 0$.]

> *Coursework*
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> Please hand in the solution to question 3 by Friday (of week in 2!). Please place them into the file in Dr. Heil's pigeonhole in the general office on the 4th floor.

[1] Any feedback to: *M.Heil@maths.man.ac.uk*