Chapter 1

Introduction

This set of notes summarises the main results of the lecture 'Elasticity' (MT4271). Please email any corrections (yes, there might be the odd typo...) or suggestions for improvement to *M.Heil@maths.man.ac.uk* or see me after the lecture or in my office (18.07).

Generally, the notes will be handed out after the material has been covered in the lecture. You can also download them from the WWW:

http://www.maths.man.ac.uk/~mheil/Lectures/Elasticity/MT4271-Elasticity.html.

This WWW page will also contain announcements, example sheets, solutions, etc.

1.1 Literature

The following is a list of books that I found useful in preparing this lecture. I've quoted the prices where I knew what they were. **It is not necessary to purchase any of these books!** Your lecture notes and these handouts will be completely sufficient.

Textbook which covers most of the material in this lecture: Gould, P.L. *Introduction to Linear Elasticity, 2nd ed.* Springer (1994) – £51

Nice (useful) review of Linear Algebra: Banchoff, T. & Wermer, J. *Linear Algebra Through Geometry, 2nd ed.* Springer (1991).

One of the classic elasticity texts: Green, A.E. & Zerna, W. *Theoretical Elasticity.* Dover (1992) – paperback reprint of the original version from Oxford University press £11.95

And another classic: Love, A.E.H. *Treatise on the Mathematical Theory of Elasticity.* Dover (1944) – paperback reprint of the original version from Cambridge University press £15.95

A beautiful little book (but out of print!): Long, R.R. *Mechanics of Solids and Fluids.* Prentice-Hall, (1961) – £11.00 (back then, presumably...). Try the library.

My own favourite elasticity book (this book saved my PhD!): Wempner, G. *Mechanics of Solids with Applications to Thin Bodies.* Kluwer Academic Publishers Group (1982) – unfortunately only available as hardback for £126!!

1.2 Preliminaries: Index notation & summation convention

- Denote vectors/matrices/tensors by their components, i.e. $\mathbf{r} = r_i$; $\mathbf{A} = A_{ij}$
- Greek indices range from 1 to 2; Latin ones from 1 to 3.
- Kronecker Delta: $\delta_{ij} = \begin{cases} 1 & \text{for} \quad i = j \\ 0 & \text{for} \quad i \neq j \end{cases}$
- Summation convention: Automatic summation over repated indices. E.g.:

Dot product: $\mathbf{a} \cdot \mathbf{b} = a_i b_i = a_k b_k$ (Dummy index!)

δ_{ij} **'exchanges' indices:** $a_i \delta_{ij} = a_j$.

Matrix-vector products: $\mathbf{A} \cdot \mathbf{x} = A_{ij} x_j = A_{im} x_m$ $\quad$ $\mathbf{A}^T \cdot \mathbf{x} = A_{ji} x_j$

No summation over indices in brackets: E.g. diagonal matrix: $diag(\lambda_1, \lambda_2, \lambda_3) = \lambda_{(i)} \delta_{(i)j}$.

- Comma denotes partial differentiation: E.g. $\frac{\partial u_i}{\partial x_j} = u_{i,j}$.
- Some differential operators in index notation:

$$\nabla \cdot \mathbf{u} = \text{div } \mathbf{u} = u_{i,i} \tag{1.1}$$

$$\nabla \phi = \text{grad}\phi = \phi_{,i} \tag{1.2}$$

$$\nabla^2 \phi = \phi_{,ii} \tag{1.3}$$