MT3271: EXAMPLE SHEET[1] X

1.) a) A hollow cylinder (internal diameter a, outer diameter b, Lamé coefficients λ, μ) is subject to a pressure p_a on the inside and a pressure p_b on the outside. There are no body forces. You can assume that the displacement field has the form $\mathbf{u} = u(r)\hat{\boldsymbol{r}}$, where $\hat{\boldsymbol{r}}$ is the unit vector in the radial direction.

Use the Navier-Lamé equations to determine the displacement field of the hollow cylinder. Thus show that the displacement field can be written in compact form as

$$u(r) = (Ap_a + Bp_b)r + \frac{Cp_a + Dp_b}{r}$$

and determine the coefficients A, B, C and D in terms of a, b, λ and μ.

b) Apply the result obtained in part (a) to the problem sketched in Fig. 1: A hollow cylinder (inner diameter a_1, outer diameter b_1 and coefficients A_1, B_1, C_1 and D_1) is surrounded by a second hollow cylinder (inner diameter $a_2 = b_1$, outer diameter b_2 and coefficients A_2, B_2, C_2 and D_2). In the unstressed state the interface pressure between the two cylinders is zero.

Show that the interface pressure p_I between the two cylinders is given by

$$p_I = p_0 \frac{A_1 a_2^2 + C_1}{(A_2 - B_1)a_2^2 + (C_2 - D_1)}$$

when the inside of the inner cylinder is subjected to a positive pressure p_0 and the outside of the outer cylinder is stress free. [Hint: the inner and outer cylinder stay in contact during the deformation].

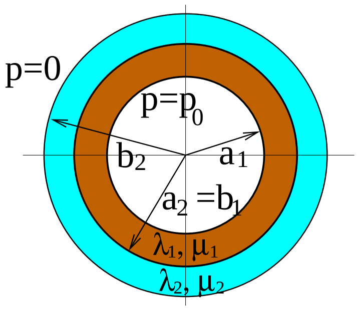

Figure 1: Sketch of two hollow cylinders.

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