# MT4271 Elasticity

**Credit rating** 10

Two lectures each week and fortnightly examples classes.

## General description

This course unit gives an introduction to the linearised theory of elasticity. A typical problem of the subject is as follows: Suppose an elastic body (e.g. an underground oil pipe) is subjected to some loading on its outer surface. What is the stress distribution which is generated throughout the body? Does this stress distribution have unexpectedly large values which might lead to failure? The subject is developed, and particular problems solved, from a mathematical standpoint.

## Aims

The course unit shows students how a physical situation could be translated into a mathematical boundary value problem and how their previous knowledge on differential equations could be used to solve the boundary value problem analytically.

## Objectives

On successful completion of the course unit students will be able to

- calculate stresses, strains and tractions, and formulate boundary value problems
- understand consitutive relations for elastic solids and compatibility constraints
- solve various two-dimensional problems (plane strain) using the Airy stress function.

## Prerequisites

**MT2121** Multiple Integrals and Vector Field Theory. **MT2202** Ordinary Differential Equations.

## Syllabus

1. Analysis of strain. The infinitesimal strain tensor; maximum normal strain. Equations of compatibility of strain.
2. Analysis of stress. The traction vector and the stress tensor; maximum normal stress. Stress equations of motion and their linearisation.
3. Stress-strain relations. Elastic and linearly elastic materials; isotropic materials. Equations of compatibility of stress for an isotropic materials in equilibrium (Beltrami-Michell equations). Navier's equation of motion for the displacement vector. Formulation of boundary value problems of linear elastostatics.
4. One-dimensional problems. A selection of soluble problems (which are effectively one-dimensional) in Cartesian, cylindrical polar or spherical polar coordinates. St. Venant's principle.
5. Plane strain problems. Theory of plane strain, Airy stress function. A selection of soluble

two-dimensional problems using plane-strain theory.

## Textbooks

The course does not follow one particular book. A good book, covering most of the course, is

P.L. Gould, *Introduction to Linear Elasticity*, 2nd Edition, Springer, 1994.
This book, and many others on the theory of elasticity can be found at 531.38 in the John Rylands University Library.

**Assessment** Two hour end of semester examination (100%).

**Lecturer** Dr Matthias Heil (Room 18.07, Telephone 5808)

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