MT4271: EXAMPLE SHEET[1] VI (for Nov. 19, 1998)

1.) Consider the three-dimensional displacement field

$$u_i = \frac{\lambda + 2\mu}{\mu(\lambda + \mu)}\ \psi_{i,jj} - \frac{1}{\mu}\ \psi_{j,ji}.$$

Provided there are no body forces, what condition does the vector function ψ_i have to fulfill for this displacement field to be a solution of the Navier-Lamé equations?

2.) What is the necessary relationship between the constants A and B if

$$\phi = Ax_1^2x_2^3 + Bx_2^5$$

is to serve as an Airy stress function?

3.) **a)** Show that

$$\phi = \frac{3F}{4c}\left(x_1x_2 - \frac{x_1x_2^3}{3c^2}\right) + \frac{P}{4c}x_2^2$$

is a valid Airy stress function.

b) Now assume that ϕ describes the stress field in the cantilever beam of thickness $2c$, as sketched in Fig. 1. Determine the stress field and consider the physical meaning of the constants F and P [Hint: Examine the resultant forces at the left end of the beam].

c) Given the physical interpretation from part (b), explain why τ_{11} increases linearly with x_1. [Hint: Examine the balance of moments about the point $(x_1, 0)$].

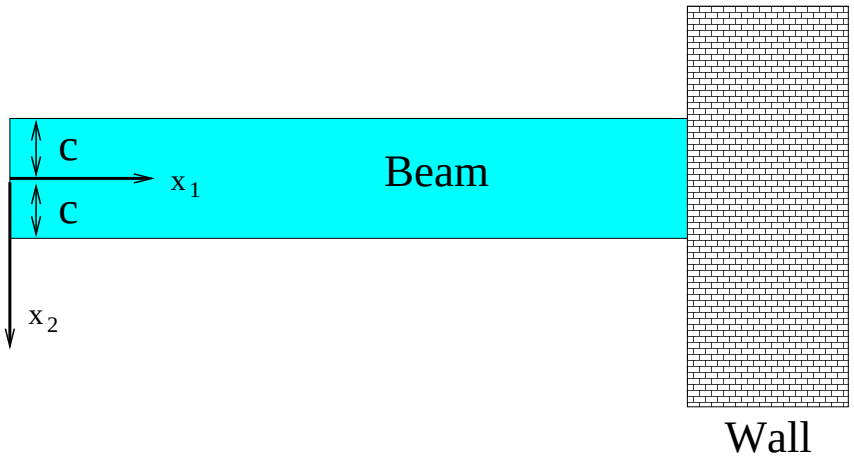

Figure 1: Sketch of a cantilever beam.

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