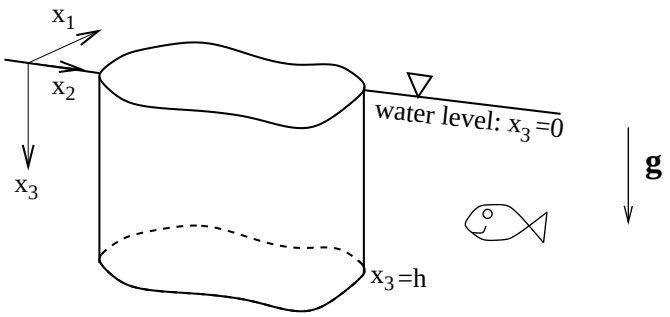
MT4271: EXAMPLE SHEET[1] V (for Oct. 29, 1998)

1.) A homogeneous isotropic linearly elastic cylinder of arbitrary cross section is immersed vertically in a stationary fluid of the same density ρ. The upper face of the cylinder is level with the free surface of the fluid and the air pressure is zero.



a) State what the tractions on the three faces (top, curved side and bottom) of the cylinder are [Hint: Compare to problem 4 on example sheet III].

b) What is the body force inside the cylinder?

c) Determine the stress field in the cylinder from the equilibrium conditions and from the stress boundary conditions. [Hint: Try the simplest linear functions for τ_{ij} which are compatible with the boundary conditions].

d) Can you be sure that the stress field corresponds to a continuous displacement field? If so, determine the displacement field, ignoring rigid body motions wherever possible.

2.) A homogeneous isotropic linearly elastic body has the form of an infinite circular cylinder of radius a whose outer edge is stress free. It is rotating about its axis with constant angular speed ω. In a coordinate system that rotates with the cylinder, the displacement $\mathbf{u}$ satisfies the Navier-Lamé equations with a body force $\mathbf{F} = \rho\omega^2 r\mathbf{e}_r$. Starting with the assumption that $\mathbf{u} = u(r)\mathbf{e}_r$ (because of the symmetry of the configuration), find $u(r)$ and the stresses in the cylinder in terms of the engineering constants E and ν.

3.) An infinite homogeneous isotropic linearly elastic body contains a spherical cavity of radius a. The cavity is filled with gas of uniform pressure p_0 and there is no body force. Find the displacement everywhere in the solid. What is the fractional increase in the radius of the cavity due to the gas pressure?

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