MT4271: EXAMPLE SHEET[1] III (for Oct. 15, 1998)

1.) The strains $e_{ij}(\mathbf{r})$ are given throughout a body. Show that the corresponding displacement field $u_i(\mathbf{r})$ is only determined to within a rigid-body displacement. Use the following two steps: (i) Show that if two displacement fields u_i^1 and u_i^2 correspond to the same strain field, then the strain field corresponding to $u_i^\Delta = u_i^1 - u_i^2$ is $e_{ij}^\Delta = 0$. (ii) Given (i), we need to show that $e_{ij} = 0$ corresponds to a rigid body displacement. [Hint: Use the fact that $e_{ij} = 0$ implies $\omega_{ij} = const.$ (see previous example sheet)].

2.) Find a displacement field corresponding to the strains

$$e_{ij} = \epsilon(a + bx_3)\delta_{ij}$$

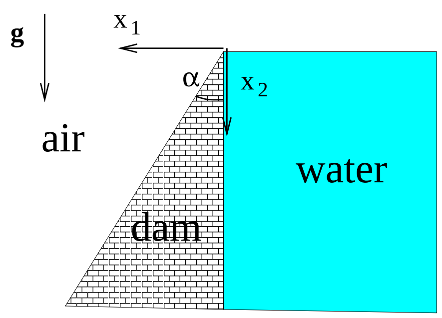
where $\epsilon \ll 1, a$ and b are constants. Note that we're not interested in the most general solution, so try dropping constants of integration whereever possible.

3.) Consider the infinitesimal tetrahedron used in the derivation of the stress tensor and show that

$$\mathbf{n}_i ds_i + \mathbf{n} ds = 0$$

where the $\mathbf{n}_i$ are the outside unit normal vectors on the three faces on which $x_i = const.$, the ds_i are their areas, $\mathbf{n}$ is the outside unit normal vector on the fourth (general) face and ds is its area.

4.) An angular dam separates water (on the right) from air (on the left). The vertical right edge of the dam is located at $x_1 = 0$. The densities of water and the dam are ρ_w and ρ_0, respectively.



The stress field in the dam has the form:

$$\tau_{11} = ax_1 + bx_2; \quad \tau_{22} = cx_1 + dx_2 - \rho_0 g x_2; \quad \text{and} \quad \tau_{12} = \tau_{21} = -dx_1 - ax_2,$$

where a, b, c and d are constants. (The problem is two-dimensional; we can assume that $\tau_{i3} = \tau_{3i} = 0$). Determine a, b, c and d from the boundary conditions. [Hints: The hydrostatic pressure on the right edge is $p = \rho_w g x_2$; the left edge is exposed to air (at pressure zero)].

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