MT4271: EXAMPLE SHEET[1] II (for Oct. 8, 1998)

1.) A 2D body occupying the region $\{d : 0 \leq x_1 \leq 1, 0 \leq x_2 \leq 1\}$ is displaced by the following displacement field (see problem 3 on the last sheet) $u_1 = \epsilon(x_1 + 2x_2)$; $u_2 = \epsilon(3 + x_2)$ where $\epsilon \ll 1$.

a) Compute the extension $e_{\mathbf{n}_1}$ of a line element in the direction of the unit vector $\mathbf{n}_1 = (3/5, 4/5)^T$ and the change in the angle between $\mathbf{n}_1$ and $\mathbf{n}_2 = (-4/5, 3/5)^T$.

b) Find the principal axes and principal strains – i.e. the directions in which the deformation is purely extensional. Check the calculated principal strains by directly computing the extensions of line elements in these directions [since the strain is spatially constant, you can simply determine the lengths of finite line elements before and after the deformation]. Relate the principal axes to the sketched deformation of the body (from the last example sheet).

2.) .

a) Show that $d = \text{div } \mathbf{u}$ and $\boldsymbol{\omega} = 1/2 \text{ curl } \mathbf{u}$.

b) Consider the rectangular parallelepiped, formed by the three infinitesimal vectors $(dx_1, 0, 0)^T, (0, dx_2, 0)^T, (0, 0, dx_3)^T$. Show that the fractional volume change of this infinitesimal volume element is given by d (as claimed in the lecture).

3.) Let $\underline{\mathbf{e}}$ be a 3x3 matrix and $\underline{\mathbf{a}}$ be a 3x3 non-singular matrix. Show that the characteristic polynomial $P(\lambda) = det(\underline{\mathbf{e}} - \lambda\underline{\mathbf{I}})$ remains unchanged if $\underline{\mathbf{e}}$ is replaced by $\underline{\mathbf{a}}\ \underline{\mathbf{e}}\ \underline{\mathbf{a}}^{-1}$. [Hint: $det(\underline{\mathbf{A}}\ \underline{\mathbf{B}}) = det(\underline{\mathbf{A}})det(\underline{\mathbf{B}})$]. Deduce that the coefficients of the characteristic polynomial are invariants under orthogonal transformation.

4.) Proof that

$$\frac{\partial \omega_{ik}}{\partial x_j} = \frac{\partial e_{ij}}{\partial x_k} - \frac{\partial e_{kj}}{\partial x_i}.$$

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