MT4271: EXAMPLE SHEET[1] I (for Oct. 1, 1998)

1.) Which one of these equations in index notation are valid? Remember the summation convention!

a) $c = a_i b_i$

b) $c = a_{ij} b_i$

c) $c_i = a_{ij} b_i$

d) $c_i = a_{ij} b_j$

e) $c_i = a_{ji} b_j$

f) $\sigma_{ij} = \alpha_{ij} T + E_{ijkl} e_{kl}$

g) $\sigma_{ij} = \alpha_{kl} T_i + E_{ijkl} e_{ij}$

h) $k_{ijkl} = a_i b_{kl} c_{njm} d_{mn} + e_{ik} e_{jn} f_{nl}$

2.) Using a comma to denote partial differentiation (e.g. $\partial u/\partial x_2 = u_{,2}$), transform the following expressions into index notation:

a) $\nabla u(x_1, x_2, x_3)$

b) $\underline{\mathbf{A}} = \nabla \mathbf{u}(x_1, x_2, x_3)$

c) $\nabla \cdot \mathbf{u}(x_1, x_2, x_3) = f(x_1, x_2, x_3)$

d) $\nabla^2 u(x_1, x_2, x_3) = f(x_1, x_2, x_3)$

e) $\nabla^2 \mathbf{u}(x_1, x_2, x_3) = \mathbf{f}(x_1, x_2, x_3)$

3.) A 2D body occupying the region $\{d : 0 \leq x_1 \leq 1, 0 \leq x_2 \leq 1\}$ is displaced by the following displacement fields:

a) $u_1 = \epsilon(x_1 + 2x_2)$; $u_2 = \epsilon(3 + x_2)$ where $\epsilon \ll 1$

b) $u_1 = U_1 - x_1 + \sqrt{x_1^2 + x_2^2}\, \cos\left(\arctan\left(\frac{x_2}{x_1}\right) + \Phi\right)$;
$u_2 = U_2 - x_2 + \sqrt{x_1^2 + x_2^2}\, \sin\left(\arctan\left(\frac{x_2}{x_1}\right) + \Phi\right)$ where U_1, U_2, Φ are constants.

Sketch the deformed body D (setting $\epsilon = 1$ in (a) for simplicity). For the displacement field (a) determine the deformation gradient tensor and the strain and rotation tensors. For the displacement field (b) determine the linear strain and rotation tensors for $\Phi \ll 1$ – Think before you calculate!

[1] Any feedback to: *M.Heil@maths.man.ac.uk*