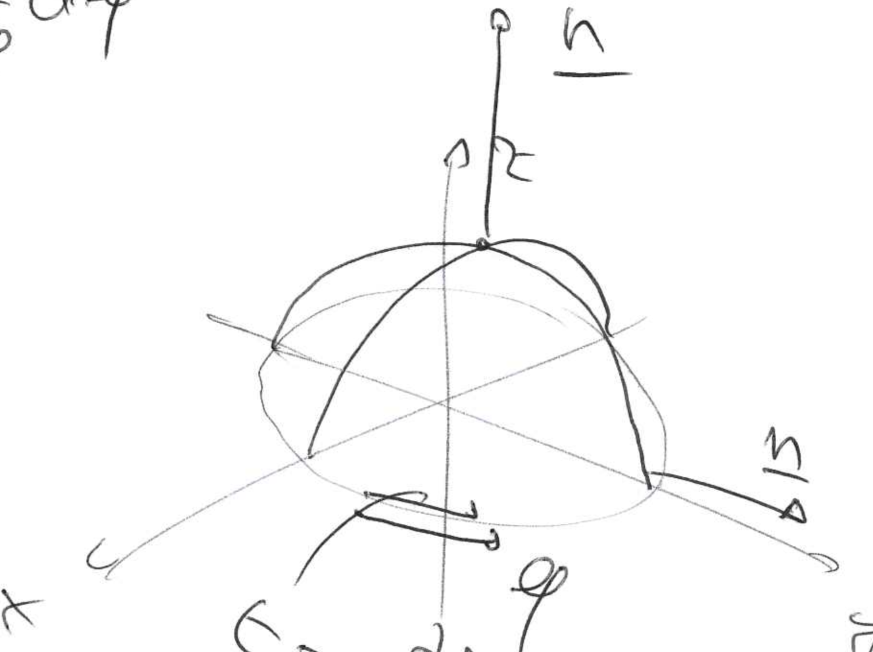
LECTURE 21
Stokes thm

$$\underbrace{\oint \underline{F} \cdot d\underline{r}} = \iint (\nabla \times \underline{F}) \cdot \underline{n} \, dS$$

$$\oint \underline{F} \cdot \frac{d\underline{r}}{d\varphi} \, d\varphi$$

Need:

$\underline{r}(\varphi)$



$\underline{t} = \frac{d\underline{r}}{d\varphi}$

The statement re the right-hand-screw rule is a topological statement.

20401 Exam

20 credit module - 3hr. exam

- ~~weighted average~~ ~~(don't get a poor mark in this and real/complex analysis)~~

Section A - 3 questions

Section B - 3 questions

4. Early material (coord. systems/chain rules/calculus several variables)

5. PDEs (First Order)

"homogeneous" - neither word nor concept will appear or be required.

Words used by Prof. Heil ~~will~~ may appear in questions.

6. Vector Calculus.

Answer 5 questions

(Revise in such away that you are prepared to answer any of the 6 questions).

Gen. form of 1st order PDEs

$$\underbrace{x}_{a}\frac{\partial u}{\partial x} \cancel{+} \underbrace{-}_{b}\frac{\partial u}{\partial y} = \underbrace{1}_{c}$$

"Dove-method"

$$\frac{dx}{x} = \frac{dy}{-1} = \frac{du}{1}$$

on char:

$$\int \frac{dx}{x} = -\int dy$$

$$\ln\frac{x}{D} = -y \, ; \Rightarrow \boxed{y = -\ln\left(\frac{x}{D}\right)}$$

char:

is family of closed curves, one curve for each D.

$$\boxed{\frac{x}{D} = e^{-y}}$$

$$\Downarrow$$

$\Leftarrow$

Along closed

$$\boxed{D = x e^{y} \text{ is constant}}$$

Along char:

$$\frac{du}{dy} = -1 \Longrightarrow u = -y + C$$

something/anything that remains const. along the charact.

$\Longrightarrow$ general soln:

$$u(x,y) = -y + f\left(xe^{y}\right)$$

arbitrary fct.

What about implicit soln?

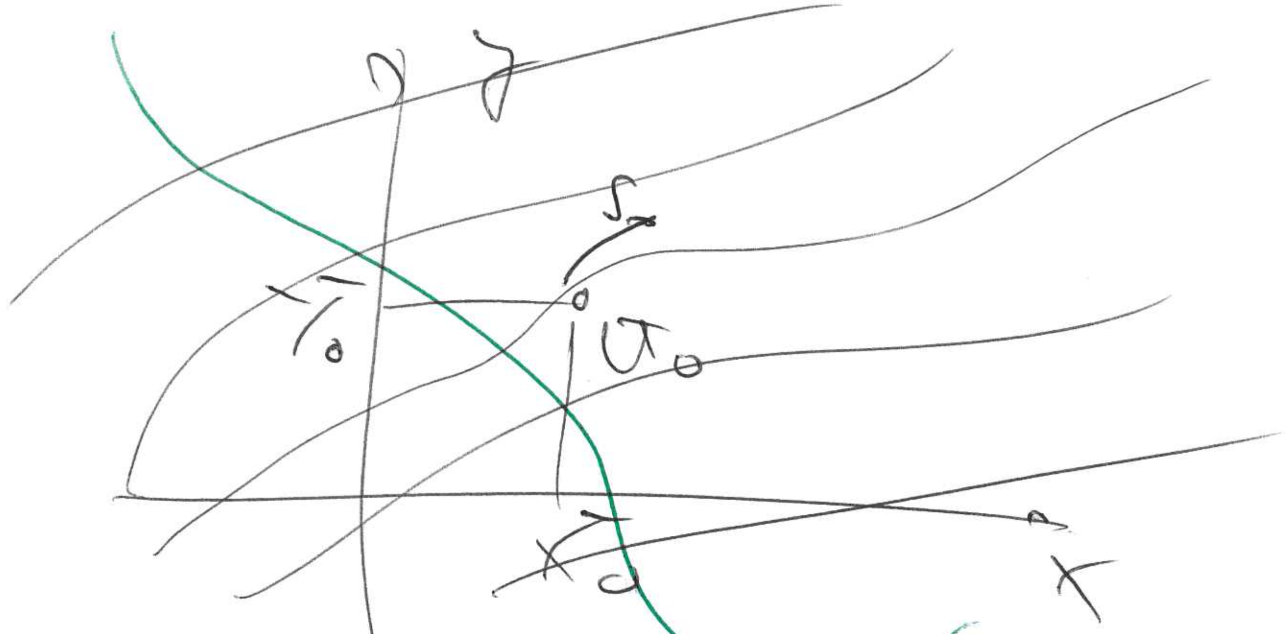
$$x\frac{\partial u}{\partial x} - \frac{\partial u}{\partial y} = 1$$

$$\begin{array}{ccc} x\frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} & 1 \\ \rightsquigarrow & \rightsquigarrow & \rightsquigarrow \\ \frac{dx}{ds} & \frac{dy}{ds} & \frac{du}{ds} \end{array}$$

$$\frac{dx}{ds} = x \Longrightarrow x(s) = Ce^{s}$$

$$\frac{dy}{ds} = -1 \Longrightarrow y(s) = D - s$$

$$\frac{du}{ds} = 1 \qquad u(s) = E + s$$



$$\boxed{\begin{aligned} x(s) &= X_0 e^{s} \\ y(s) &= Y_0 - s \\ u(s) &= U_0 + s \end{aligned}}$$