2M1 – Q-STREAM: EXAMPLE SHEET[1] II

1. **Solution of PDEs "by inspection"**

   (a) Show that the function $u(x, y) = x - y$ is a solution of the PDE

   $$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 0.$$

   (b) Show that $u(x, t) = \sin(x + t) + \cos(x - t)$ is a solution of the 1D linear wave equation

   $$\frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial t^2}$$

   [**Hint:** Remember the chain rule!]

   (c) Determine the values of the constants a and b for which the function $u(x, t) = e^{at}(\sin x - bx^2)$ satisfies the 1D unsteady heat equation

   $$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}.$$

2. **Separation of variables for the 1D linear wave equation**

   Use separation of variables to solve the 1D linear wave equation

   $$\frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial t^2}$$

   subject to the boundary conditions

   $$u(x = 0, t) = 0 \quad \text{and} \quad u(x = 1, t) = 0$$

   and the initial conditions

   $$u(x, t = 0) = 0 \quad \text{and} \quad \left.\frac{\partial u}{\partial t}\right|_{t=0} = \sin(3\pi x).$$

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3. **Separation of variables for the 1D unsteady heat equation**

Use separation of variables to solve the 1D unsteady heat equation

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$$

subject to the boundary conditions

$$u(x=0,t)=0 \quad \text{and} \quad u(x=\pi,t)=0$$

and the initial condition

$$u(x,t=0)=\sin(x).$$

<u>**Hint:**</u>
Choose the sign of the separation constant so that the ODE for $T(t)$ becomes $dT/dt + \omega^2 T = 0$, say, where $\omega^2 > 0$. This choice is motivated by the physics of the problem. Explain why!