2M1 – Q-STREAM: EXAMPLE SHEET[1] I

1. **Partial derivatives**

   Find the partial derivatives $\partial f/\partial x$, $\partial f/\partial y$, $\partial^2 f/\partial x^2$, $\partial^2 f/\partial y^2$ and $\partial^2 f/(\partial x \partial y)$ for the following functions:

   (a) $f(x, y) = x^2 - 2xy + 6x - 2y + 1$
   (b) $f(x, y) = \exp(xy)$
   (c) $f(x, y) = x^2 + y^2 + x^2 y + 4$

   In each case confirm that

   $$\frac{\partial}{\partial x}\left(\frac{\partial f}{\partial y}\right) = \frac{\partial}{\partial y}\left(\frac{\partial f}{\partial x}\right).$$

   Hint: Don't forget the chain and product rules:

   $$\frac{d}{dx} f(g(x)) = f'(g(x))\; g'(x)$$

   and

   $$\frac{d}{dx}\Big(f(x)\; g(x)\Big) = f'(x)\; g(x) + f(x)\; g'(x).$$

2. **Stationary points**

   Determine the stationary points of the three functions in the previous question and classify them.

3. **Taylor series**

   Show that the Taylor series expansion of $f(x, y) = e^{xy}$ about the point $(2, 3)$ is

   $$f(2 + \epsilon, 3 + \delta) = e^6 \left[1 + 3\epsilon + 2\delta + \frac{1}{2}\Big(9\epsilon^2 + 14\epsilon\delta + 4\delta^2\Big)\right] + \cdots,$$

   or, if you prefer the alternative notation:

   $$\begin{aligned} f(x, y) &= e^6 \Big[1 + 3(x-2) + 2(y-3) + \\ &+ \frac{1}{2}\Big(9(x-2)^2 + 14(x-2)(y-3) + 4(y-3)^2\Big)\Big] + \cdots, \end{aligned}$$

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