Dimensional Adjectives and Measure Phrases in Vector Space Semantics

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The paper presents a compositional semantics for absolutes and simple comparatives of dimensional adjectives (e.g. *short*, *taller than*) within the framework of vector space semantics.The analysis is extended to include measure phrases.

1 Introduction

This paper presents a vector space semantics (VSS) for constructions involving dimensional adjectives (DAs)[1] in predicative position of the kind exemplified in (1).

(1) a. Sam is (6cm) taller than Jo.

b. The board is (1m) long.

(1)a. is an example of a simple comparative, (1)b. exemplifies the so-called absolute construction.

VSS was developed by Zwarts (1997) and Zwarts and Winter (1997) to account for phenomena in the spatial domain. One of the main goals

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[1]DAs are a subgroup of gradable adjectives. Gradable adjectives (GA) are those adjectives that order objects with respect to the extent to which an object possesses the relevant property, e.g. *intelligent, big*. With dimensional adjectives this extent can be quantified using measure phrases, e.g. 3cm long.

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of the paper is to show that it is a versatile framework, which can also successfully be applied to other semantic areas such as GAs, and offer new insights within those domains.

In the development of formal semantic theories it is common practice to postulate different ontological objects to account for different phenomena, e.g. (sets of) *points* for spatial, *events* for temporal, *degrees* for comparative relations etc. However, by doing so one can easily miss the similarities that hold between constructions in these different domains. For instance, typologists have established that many languages employ the same syntactic devices to encode spatial, temporal and comparative relations, and in some languages this even extends to causal relations (Stassen, 1984). One such language is Eskimo, which uses the ablative case -*mit*(sg.)/*nit*(pl.), for spatial, temporal, causal, and comparative relations (see (2); Mey (1976)):

(2) a. igdlumi**nit** anivoq [spatial]
‘he went out of his (own) house’
b. tássanîlerqârning**mit** [temporal]
‘from the time you first started being here’
c. mikiner**mit** támarpoq [causal]
‘it was lost because it was so little’
d. amaroq qingmi**mit** agneruvoq [comparative]
‘the wolf is bigger than the dog’

Syntactic parallelism can often reflect semantic parallelism. In Eskimo this seems to be the case. According to Mey (1976) “[t]he all-pervading characteristic of the Eskimo ablative is the concept of distance (static or dynamic; either in place or in time).” This kind of similarity can more easily be captured within a framework that uses the same ontological primitives. As will become clear below, the notion of distance between two objects plays a central role in the VSS of comparatives, as well as in the VSS of locative prepositions (Zwarts, 1997; Zwarts and Winter, 1997). Thus, Mey’s observation can directly be modelled in VSS, and the Eskimo ablative can uniformly be analyzed as making reference to the same kind of object. A framework that postulates different primitives for each domain will have to generalize over distances between different kinds. In general, it seems to be advantageous to use the same ontological objects to study the semantic similarities between domains.

VSS, a framework in the tradition of Montague grammar that adds vectors to the standard ontology, is a very promising framework in this respect, since vectors are abstract enough objects to be useful across a variety of domains. An example of how similarities across domains can be explained in a uniform way within VSS is the Modification Condition MC. The MC explains the (in)felicity of MPs with certain locative prepositions, *10m outside/*inside the house* (Zwarts and Winter, 1997), and DAs, *2m tall/*short* (see work in progress by Winter and section 2.3).

Furthermore, the ontology of vector spaces provides a very powerful mechanism, which should be sufficient to analyze linguistic expressions which require a multi-dimensional static or dynamic analysis.[2]

A long-term goal in developing VSS is to build a framework that facilitates the investigation of similarities between domains. We hope to show in this paper that VSS can straightforwardly be extended to the domain of GAs, and investigate in future research its applicability to temporal and causal relations.

Section 2 presents a VSS semantics for simple comparatives such as (1)a. and absolutes such as (1)b., which is then extended in section 2.3 to include measure phrases (MPs) in comparatives (*Sam is 3cm taller than Jo*). It will be shown that the semantics developed in section 2 together with the semantics for MPs presented in Zwarts (1997) makes the incorporation of MPs straightforward. We conclude, in section 5, by indicating some directions for future work.

2 Analysis of Comparatives and Absolutes

Comparatives and absolutes present a variety of interesting semantic puzzles, for example with respect to their interaction with logical connectives, quantifiers, opaque contexts, and their monotonicity properties (von Stechow, 1984), which have given rise to a considerable literature. Many of these will not be discussed in this paper, but we hope to investigate in future work how VSS can contribute to their solution. Here, we will focus on laying the groundwork and illustrate how MPs receive a natural analysis.

Two major approaches to the analysis of GAs can be distinguished:

[2]The semantics for DAs presented here makes use of only a relatively small number of the features provided by the ontology of vector spaces, and it might therefore be argued that this framework comes with too much unnecessary formal baggage. While this might be a valid criticism for the semantics of simple DAs, it should be kept in mind that the full power of vector spaces might be needed for other domains. For example, a VSS analysis of directional prepositions will arguably have to make use of the dynamic nature of vectors (Johan van Benthem, p.c.). An integrated analysis of different domains within a single framework thus requires a formal apparatus powerful enough to handle the most complex cases. Future research will have to show whether any linguistic phenomenon requires the full complexity of a vector space ontology.

degree-based approaches, e.g. Bierwisch 1984; Cresswell 1976; Hellan 1981; Kennedy 1997; Seuren 1973; von Stechow 1984, and so-called delineation approaches, e.g. Lewis 1970; Klein 1980; Klein 1991. The present account can be considered a degree-based approach, and we will briefly give the basic intuitions underlying degrees in the following section. Our VSS analysis bears a close resemblance to Bierwisch (1984), but there is at least one fundamental difference which will be discussed in 3.1. As a representative of delineation approaches, Klein's analysis (Klein, 1980, 1991) will be discussed in section 3.2.

2.1 Degree-based Approaches

The basic intuition underlying degree-based approaches is that individuals can possess a property to a certain measurable degree, for example a person can be 1.70m tall. This intuition is generally captured by analyzing GAs as relations or functions between individuals and degrees, where these degrees are generally conceptualized to form a scale associated with the dimension referred to by the DA. A partial ordering is imposed on degrees, on the basis of which comparatives are evaluated. For example, (1)a. is true iff the degree $\mathbf{d_1}$ to which Sam is tall is greater than the degree $\mathbf{d_2}$ to which Jo is tall: $\mathbf{d_2} < \mathbf{d_1}$. Absolutes are generally analyzed as comparing an individual with a contextually determined standard $\mathbf{s}$, such that (1)b. is true iff the degree $\mathbf{d}$ to which the board is long is greater than $\mathbf{s_l}$, the standard for length: $\mathbf{s_l} < \mathbf{d}$.

It turns out that MPs are most easily incorporated in accounts that represent the comparison relation in a slightly more complex manner, introducing a *difference* degree, $\mathbf{d_3}$ and a concatenation operation '+' (Bierwisch, 1984; Hellan, 1981; von Stechow, 1984). In such a system, (1)a. is true iff $\exists \mathbf{d_3}[\mathbf{d_2} + \mathbf{d_3} = \mathbf{d_1}]$. An MP simply specifies the value of this difference degree as for example 6cm in (1)a. von Stechow (1984) argues that such a move is necessary in order to account for MPs. While this is not necessarily true (see e.g. Klein's account discussed in section 3.2 for an alternative solution), it is certainly a very intuitive approach to the MP problem.[3]

Bierwisch's (1984) difference degrees are furthermore directional. Thus $\mathbf{d_3}$ can start at $\mathbf{d_2}$ and end at $\mathbf{d_1}$, or the other way round. This feature accounts in a straightforward manner for the contrary nature of antonymic adjectives such as *long* and *short*, i.e. *Sam is shorter than*

[3]Kennedy (1997) uses extents together with a concatenation operation rather than degrees. The description of his approach would lead us to far afield, but we would like to point out that the incorporation of MPs should also be possible in his system, though he does not develop an explicit analysis.

Jo is true iff $\exists \mathbf{d_3}[\mathbf{d_1} \Leftrightarrow \mathbf{d_3} = \mathbf{d_2}]$ (with d_3 being positive). Obviously, *Jo is shorter than Sam* $\Leftrightarrow$ *Sam is taller than Jo*. Bierwisch (1984) shows that previous analyses cannot account for this equivalence or even the fact that *short* and *tall* make reference to the same dimension, without stipulating additional rules.

2.2 VSS Analysis

We use the same vector space ontology as Zwarts (1997) and Zwarts and Winter (1997), and refer the reader to those works for formal definitions.[4]

Informally, a vector space is a set of vectors V over the real numbers, together with the operations vector addition $+$ and scalar multiplication $\cdot$, which are defined in the usual way. The vector norm $||$ determines the length of a vector (Zwarts and Winter, 1997). In addition to simple vectors, we make use of *located vectors* $\mathbf{u}$ which are members of $V \times V$, s.t. $\mathbf{u} = < w, v >$. The start point vector of $\mathbf{u}$ is w, the end point vector is defined as $w + v$ (see Fig. 2.2 (Zwarts, 1997)). We will use the functions *spo* and *epo* to map a located vector $\mathbf{u}$ onto it's start and end point vectors respectively. Simple vectors are located vectors whose starting point vector is the zero vector. Conventionally, the letters v, w, u are used for variables for simple vectors, bold face **v**,**w**,**u** for located vectors, and V, W for sets of vectors.

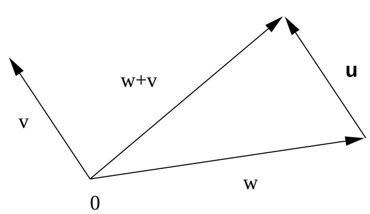

Figure 12.1: located vectors

DAs make reference to dimensions such as HEIGHT. Following Winter (1999) we assume for each dimension D a *unit vector* u_d s.t. $|u_d| = 1$.[5] An adjective root such as *tall-* or *short-* denote sets of located vectors $\mathbf{v}=< r_1 u_h, r_2 u_h >$, s.t. the vectors in the set denoted by a positive adjective have the same direction as the unit vector, and those in the

[4] See also for example Lang (1971) for an introduction to linear algebra.

[5] We can think of each unit vector as corresponding to a different dimension in a multidimensional vector space.

denotation of a negative adjective point in the opposite direction (see Fig. 2.2. For better visibility, the *pos* and *neg* vectors are displayed below u.)

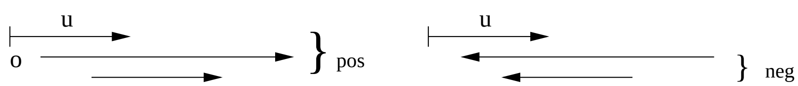

Figure 12.2: positive and negative DAs

For the purposes of this paper, we assume that a function *dim* maps individuals onto a set of *d(imension)-vectors*. A d-vector is a simple vector $r \cdot u_d$ s.t. r equals the extent to which an individual possesses the property referred to by the relevant DA.

On the basis of these notions we can now analyze comparatives and absolutes as follows. Consider example (1)a. *Jo* is mapped onto its set of d-vectors, with j_h in Fig.2.2a. being in that set. The phrase *taller than Jo* is then analyzed as denoting the set of located vectors V starting at the endpoint of j_h. The set V is the output of the comparative morphemes *more/-er* and *less*, which are analyzed as two-place functions that take a DA root meaning and the set W denoted by the *than*-clause as arguments, and map them onto the set of vectors V, where *more/-er* preserves the directionality of the vectors in the set denoted by the DA root, and *less* reverses it. Thus, in Fig.2.2a. $\mathbf{m}$ is a member of the set denoted by *taller than Jo*, and $\mathbf{l}$ is a member of the set denoted by *less tall than Jo*. For *short*, directions are reversed.

Figure 12.3: taller/less tall than Jo
(s_h, j_h: height of Sam/Jo, $\mathbf{l}, \mathbf{m}$: difference vectors)

The subject of predication, *Sam*, is also mapped onto a set containing its d-vectors. There will be exactly one vector $\mathbf{m}$ in the set denoted by *taller than Jo* such that its endpoint coincides with the endpoint of s_h. This is illustrated in Fig.2.2b. The semantics for (1)a. will express that there exists a vector $\mathbf{m}$, such that $j_h + \mathbf{m} = s_h$.

Unlike comparatives, absolute DAs are analyzed as sets of d-vectors rather than difference vectors, mainly for technical reasons (see section

2.3). Intuitively, a board is long/short if it exceeds/is below a contextually determined standard s_l. [6] The d-vectors in the set denoted by the absolute DA will therefore be required to be longer/shorter than the standard vector s. The final representation for (1)b. will express that there exists a vector $\mathbf{u}$ such that $\mathbf{u} + s = b_l$.

Formal Derivation

This section presents the formal definitions of the various elements participating in a comparative relation, and the compositional derivation of the examples in (1.) In contrast to accounts that derive comparatives from absolute DAs, we follow the analysis in Kennedy (1997) and derive both forms from a common DA root.

The denotations of the DA roots *tall-* and *short-* are:

(3) a. $\mathbf{tall}\text{-} \stackrel{\text{def}}{=} \lambda\mathbf{v}.\mathbf{v} = < r_1 u_h, r_2 u_h > \ \wedge r_1 + r_2 > r_1$
b. $\mathbf{short}\text{-} \stackrel{\text{def}}{=} \lambda\mathbf{v}.\mathbf{v} = < r_1 u_h, r_2 u_h > \ \wedge r_1 + r_2 < r_1$

For simplification of the semantic representations, we will make use of the notion of a scale introducing scale predicates which hold of the union of the sets of vectors denoted by a positive DA and its antonym. Thus, HEIGHT refers to the scale associated with the dimension of height.[7]

We furthermore introduce the two predicates *pos* and *neg* s.t. *pos* is true of a vector $< r_1 u_d, r_2 u_d >$ iff $r_1 + r_2 > r_1$, and *neg* is true of a vector $< r_1 u_d, r_2 u_d >$ iff $r_1 + r_2 < r_1$. The simplified representations are given in (4)

(4) a. $\mathbf{tall}\text{-} \stackrel{\text{def}}{=} \lambda\mathbf{v}.\text{HEIGHT}(\mathbf{v}) \wedge pos(\mathbf{v})$
b. $\mathbf{short}\text{-} \stackrel{\text{def}}{=} \lambda\mathbf{v}.\text{HEIGHT}(\mathbf{v}) \wedge neg(\mathbf{v})$

As mentioned above, we assume a function *dim* which maps an individual x onto its set of d-vectors $dim_{<D,x>}$ for all dimensions D. Since

[6] We will not discuss how these standards are obtained, see Kennedy (1997) for discussion. Also, the standard for a negative adjective can in principle be different from that of a positive adjective.

[7] Note that as Yoad Winter points out, the additional notion of a scale is not strictly necessary to account for the problems dealt with in this paper given the underlying vector space ontology. However, as scales are intuitive notions, and as their use results in simpler formulae, we will make use of them in this paper. We would also like to point out that the resulting scales are not *canonical* in Bierwisch's (1984) sense, in which a canonical scale does not have a negative extension. The VSS denotations of *tall-* and *short-* do contain vectors below 0. This feature is needed to account for MP modification in the present analysis (see endnote 13, section 2.3).

in this paper we are only dealing with comparatives of the form *x is more DA than y*, we can simply identify the denotation of *than* with *dim*.[8]

The comparative morphemes have the following interpretations:

(5) a. $\mathbf{more}' \stackrel{\text{def}}{=} \lambda \mathrm{W}.\lambda \mathbf{V}.\lambda \mathbf{v}.W(\mathbf{v}) \wedge \exists w[V(w) \wedge w = spo(\mathbf{v})]$ [9]

b. $\mathbf{less}' \stackrel{\text{def}}{=} \lambda \mathrm{W}.\lambda \mathbf{V}.\lambda \mathbf{v}.W(\bar{\mathbf{v}}) \wedge \exists w[V(w) \wedge w = spo(\mathbf{v})]$ where $\bar{\mathbf{v}}$ is defined as follows: Let $\mathbf{v}$ be the located vector $\langle r_1 u_d, r_2 u_d \rangle$, then $\bar{\mathbf{v}} = \langle r_1 u_d, \Leftrightarrow r_2 u_d \rangle$. This effects the reversal of directionality of the vectors in the DAs denotation with *less*.

The comparative morphemes thus denote sets of vectors on the scale associated with the DA whose starting point is a d-vector in the set V provided by the *than*-phrase. That the right d-vector is chosen is guaranteed by the fact that only one can have its *spo* be identical with that of a vector on the given scale.

Applying $\mathbf{more}'/\mathbf{less}'$ to $\mathbf{tall}'$ and $\mathbf{than}'(\mathbf{Jo}')$, we get the following semantics for *taller/less tall than Jo*:

(6) a. $\mathbf{more}'(\mathbf{tall}')(\mathbf{than}'(\mathbf{Jo})) = \lambda \mathbf{v}.\text{HEIGHT}(\mathbf{v}) \wedge pos(\mathbf{v}) \wedge dim_{<H,Jo>} = spo(\mathbf{v})$

b. $\mathbf{less}'(\mathbf{tall}')(\mathbf{than}'(\mathbf{Jo})) = \lambda \mathbf{v}.\text{HEIGHT}(\bar{\mathbf{v}}) \wedge neg(\mathbf{v}) \wedge dim_{<H,Jo>} = spo(\mathbf{v})$

The next step in the compositional process has to be to predicate this expression of the subject, *Sam* in the running example. This is not directly possible, since *taller than Jo* denotes a property (set) of vectors, and *Sam* is an individual not a vector. We introduce an "anti-dimension" function dim^-, which takes a set of vectors and returns a

[8] As one reviewer pointed out this will not work for more complicated sentences like (i), which is ambiguous between the readings in (ii) and (iii).

(i) The producer wants the film to be more violent than the director.

(ii) ... than the director is violent

(iii) ... than the director wants it to be violent.

Obviously, simply mapping the director onto the scale of violence will miss the reading in (iii). Ellipsis of various kinds in the comparative clause and the reconstruction of the understood elements is a hard problem, and we will not make any attempt at contributing to its solution in this paper. Furthermore, the mapping of *than*-clauses cannot be dealt with in the same way as that of *than*-phrases. For example, even if we assume that (i) has been reconstructed to (ii), the proposed *dim* could not be applied. Faller (1998a) proposes different kinds of *dim* functions to deal with some types of *than*-clauses. This is an area that clearly requires more research.

[9] We assume throughout the paper that *-er* and *more* get the same interpretation.

set of individuals s.t. one of the individuals' d-vectors has the same *epo* as the vectors in the set denoted by a comparative DA plus *than*-clause. This function is parallel to loc^- used by Zwarts and Winter (1997).

$$(7)\ \ dim^- \stackrel{\text{def}}{=} \lambda W.\lambda x.\exists \mathbf{v} \exists w.W(\mathbf{v}) \wedge w \in dim(x) \wedge epo(\mathbf{v}) = w \wedge |\mathbf{v}| > 0$$

Neither loc^- nor dim^- are associated with a particular lexical item. Instead, their application is triggered by the type mismatch between the subject of predication and the predicate. At this point one might ask why we need these functions at all, since one could just as well interpret the comparative morphemes as relations between vectors and have them type-shift both individuals. It will become clear in section 2.3 that this complication is necessary, because MPs have to have access to the set of vectors denoted by *taller than* x. Applying dim^- to **taller than Jo**′ and **Sam**′ gives us:

$$(8)\ \ \exists \mathbf{v}[\text{HEIGHT}(\mathbf{v}) \wedge pos(\mathbf{v}) \wedge dim_{<H,Jo>} = spo(\mathbf{v}) \\ \wedge\ dim_{<H,Sam>} = epo(\mathbf{v}) \wedge |\mathbf{v}| > 0]$$

This can be simplified to (9)a., where s_h and j_h stand for the d-vectors of *Sam* and *Jo* respectively. The interpretation for *Jo is shorter than Sam* is given in (9)b.

(9) a. $\exists \mathbf{v}[s_h = j_h + \mathbf{v}$[10] $\wedge pos(\mathbf{v}) \wedge |\mathbf{v}| > 0]$
b. $\exists \mathbf{v}[j_h = s_h + \mathbf{v} \wedge neg(\mathbf{v}) \wedge |\mathbf{v}| > 0]$

It can easily be seen that (9)a. and (9)b. are logically equivalent, which captures the already mentioned equivalence between *Sam is taller than Jo* and *Jo is shorter than Sam.*

Absolute DAs compare their argument with a contextually determined standard s. This standard is introduced by a phonologically empty absolute morpheme $\emptyset$ that applies to a DA root (see also Kennedy (1997)). $\emptyset$ has the following interpretation:

$$(10)\ \ \emptyset' \stackrel{\text{def}}{=} \lambda W.\lambda v.\exists \mathbf{w}[W(\mathbf{w}) \wedge v = epo(\mathbf{w}) \wedge s = spo(\mathbf{w}) \wedge |\mathbf{w}| > 0]$$

[10] A located vector $\mathbf{u}$ can be added to another vector $\mathbf{v}$, if $spo(\mathbf{u}) = epo(\mathbf{v})$. The sum vector $\mathbf{w}$ is defined by spo($\mathbf{w}$)=spo($\mathbf{v}$) and epo($\mathbf{w}$)=epo($\mathbf{u}$)(Kowalski, 1967). As simple vectors v are located vectors whose startpoint is 0, this also defines addition of a located vector to a simple vector, as is the case in the example, and in all further cases of vector addition involving located vectors in this paper. As an aside, note also that addition of located vectors is not commutative.

$\emptyset$ maps a DA onto a set of d-vectors v s.t. their endpoint defines the endpoint of a difference vector $\mathbf{w}$ whose startpoint is s. The directionality of $\mathbf{w}$ is determined by DA, which forces v to be longer/shorter than s for positive/negative DAs.

We compute the representation for (1)b. as follows.

(11) $\emptyset(\mathbf{long}\Leftrightarrow')' = \lambda v.\exists \mathbf{w}[\textsc{length}(\mathbf{w}) \wedge pos(\mathbf{w}) \wedge v = epo(\mathbf{w}) \wedge s = spo(\mathbf{w})]$

$\emptyset(\mathbf{long})'$ and $\mathbf{board'}$ are the arguments of $dim\Leftrightarrow$ which outputs (12)a. as the final interpretation for (1)b (b_l stands for the length d-vector of *board*).

(12) $\exists \mathbf{w}[b_l + \mathbf{w} = s \wedge pos(\mathbf{u}) \wedge |\mathbf{u}| > 0]$

2.3 Incorporating Measure Phrases

This section discusses sentences of the form :

(13) a. Sam is 3cm taller than Jo.
b. The board is 100cm long.

As mentioned in section 2.1, degree-based accounts that relate the two degrees associated with the two compared objects directly, have difficulties incorporating MPs. The problem is that they would somehow have to calculate the difference between the two degrees, i.e. the MP would have to have access to the denotations of both compared objects. Given the standard syntactic analysis of adjective phrases ([MP [[more [ADJ]] XP]], where XP is the (extraposed) *than*-phrase), this is not straightforward to implement compositionally.

Accounts which make explicit use of difference degrees can more easily incorporate MPs. For example, in von Stechow (1984) an MP is a function that takes a set of difference degrees as its argument. This is, as we will see below, not unlike the VSS analysis of MPs, with the difference that von Stechow first raises MPs and then quantifies them in in standard Montague fashion, so that the output of an MP is actually a proposition. Bierwisch (1984) also uses difference degrees, and as we will see in section 3.1, in his approach an MP fills actually one of the argument places of an DA.

What we aim to show here, is that the VSS semantics for MPs modifying PPs in Zwarts (1997); Zwarts and Winter (1997) carries easily over to MPs modifying DAs.

As illustrated in the previous section, the phrase *taller than Jo* denotes a set of difference vectors V between the two compared objects.

Therefore, there is no need to access the denotation of the subject. An MP in VSS is simply defined as a function that maps a set of vectors V onto a subset $W \subseteq V$ s.t. the members of W have a specific length, e.g. *3cm* in (13)a.:

(14) $\mathbf{3cm}' \stackrel{\text{def}}{=} \lambda W.\lambda \mathbf{v}.W(\mathbf{v}) \wedge |\mathbf{v}| = 3cm$

Applying $\mathbf{3cm}'$ to *taller than Jo* results in:

(15) $\lambda \mathbf{v}.\text{HEIGHT}(\mathbf{v}) \wedge pos(\mathbf{v}) \wedge dim_{<H,Jo>} = spo(\mathbf{v}) \wedge |\mathbf{v}| = 3cm$

The final semantic representation for (13)a. with the MP is:

(16) $\exists \mathbf{v}[s_h = j_h + \mathbf{v} \wedge |\mathbf{v}| = 3cm]$

In the present VSS analysis, absolute DAs denote sets of d-vectors, and intuitively an MP requires these d-vectors to be a certain length. This is the reason for not analyzing absolute DAs as sets of difference vectors. As with comparatives, an MP maps the set of d-vectors denoted by the absolute DA onto a subset s.t. its members have the length specified by the MP. However, absolute constructions present a further complication. Given the present semantics of the absolute morpheme $\emptyset$, the d-vectors would additionally be required to be longer/shorter than the standard s. However, absolutes containing MPs make no reference to a standard. A board that is 100cm long is not necessarily long compared to $\mathbf{s}_l$. We will assume that s is 0 in this case.[11]

A further property of MP modification of absolute DAs is that only positive DAs can be modified:[12]

(17) a. The board is 100cm long
b. # The board is 100cm short.

[11] Alternatively, one can introduce a second interpretation for the absolute morpheme that does not introduce a standard (this is similar to the analysis in Kennedy (1997)), but simply returns the d-vectors in the set denoted by the adjective:

(i) $\emptyset' \stackrel{\text{def}}{=} \lambda W.\lambda v.W(v)$

This is compositional, the MP can directly apply to the set denoted by *tall-$\emptyset$* and specify the length of v as before. However nothing prevents the first absolute morpheme from applying, and it is thus predicted that (13)b. is ambiguous between a reading where there is no reference to s, and a reading that requires the board to be longer than s. The second reading is however not available.

[12] See section 3.1 for how Bierwisch (1984) accounts for these data. We would like to point out that Kennedy (1997) can also account for the data in 17. In his analysis, negative DAs denote unbounded extents in contrast to positive DAs, and MPs an only apply to bounded extents. Von Stechow (1984) does not discuss negative adjectives at all, and it is not clear how he would account for this data.

A similar restriction holds for locative preposition pairs such as *inside/outside* (Zwarts and Winter, 1997):

(18) a. The tree is 10m outside the house
b. # The tree is 10m inside the house.

Zwarts and Winter (1997) account for the data in (18) on the basis of the monotonicity properties of the vector sets denoted by *outside/inside the house*. A set of vectors is said to be upward monotone (VMON$\uparrow$) iff it is closed under lengthening of its members, and downward monotone (VMON$\downarrow$) iff it is closed under shortening. MP modification of PPs is then restricted to conform to the following modification condition (MC) (Zwarts and Winter, 1997):

(19) A structure denoting a set of located vectors W can be modified by a measure phrase iff W is VMON$\uparrow$, VMON$\downarrow$ and non-empty.

Since the vectors in the set denoted by *outside the house* have their startpoints at the border of the space associated with the house and point away from it, the set is closed under both lengthening and shortening, and the MC is met. The vectors in the set denoted by *inside the house* on the other hand have their endpoints in the house so to speak. Their lengthening will result in vectors that are no longer in the set denoted by *inside the house*, and MC is not met (Zwarts and Winter, 1997).

In current work in progress, Yoad Winter (p.c.) extends this condition to MP modification of DAs. While the technical details remain to be worked out, MC can also account for both the data in (17) and the fact that the standard is required to be zero in (17)a. As Winter's (1999) analysis of absolute DAs diverges from ours, the following explanations for why the MC is or is not met differ slightly from his.

Recall that $\emptyset'(\mathbf{tall}')$ denotes a set of d-vectors which are longer than the standard s. It is clear that this set is closed under lengthening: lengthening of a d-vector longer than s will yield another d-vector longer than s. However, shortening of a d-vector will not, *unless* the standard is in fact 0. $\emptyset'(\mathbf{short}')$ on the other hand, is closed under shortening (a d-vector shorter than one that is shorter than s is also shorter than s), but for no value of s including 0 is it closed under lengthening and non-empty. Therefore an MP can modify **tall**$'$, provided the standard is zero, but not **short**. The MC also accounts for the permissibility of MPs with comparatives of both positive and negative DAs: both *taller than x* and *shorter than x* are VMON$\uparrow$ and VMON$\downarrow$ when non-empty.[13]

[13]Note that *shorter than x* is only VMON$\uparrow$ because it also contains vectors whose endpoints are below 0. This is the technical reason mentioned in endnote 7 for

3 Differences with Previous Work

The VSS analysis of DAS developed in the previous sections is in certain respects quite similar to existing degree-based approaches, in particular to Bierwisch (1984). Nevertheless there are some fundamental differences, which are discussed in 3.1. As an example of a delineation approach Klein's account (1980, 1991) will be discussed.

3.1 Bierwisch

The basic ideas behind Bierwisch's approach have already been presented in section 2. What distinguishes it from other degree-based approaches is that his difference degrees are directed, which allows him to capture the converse relation that holds between the comparatives of DAs antonyms. The VSS account differs from Bierwisch's on ontological grounds: the primitives are vectors, not degrees. Obviously, the notion of directed degrees is quite similar to that of vectors, hence the resemblance between the two accounts.

There is one aspect of Bierwisch's analysis that is fundamentally different from our VSS account, which will discuss in the following. The difference is illustrated with data concerning MPs in absolute constructions.

Bierwisch (1984) analyzes GAs as subcategorizing for a subject x and a degree argument c, i.e. they are relations between an individual and a degree as in other degree-based approaches. However, this degree is *not* the degree associated with x, but rather corresponds to the difference degree as can be seen in the following representations for positive and negative absolute GAs:

(20) a. pos-adj $\stackrel{\text{def}}{=} \lambda c.\lambda x.[\text{QUANT MAX } x] \supset [v[+c]]$
b. neg-adj $\stackrel{\text{def}}{=} \lambda c.\lambda x.[\text{QUANT MAX } x] \subset [v[\Leftrightarrow c]]$ [14]

Here, MAX specifies the (maximal) extension of x along a certain dimension and QUANT maps this value onto a degree interval beginning at 0 on the appropriate scale. Thus, QUANT MAX(x) roughly corresponds to

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allowing scales to have a negative extension. We are currently working on an analysis that maintains canonical scales and the MC by defining vector spaces to range over R^+ only. The comparatives of negative DAs would still be VMON$\uparrow$ since the vector domain is dense. The technical consequences of this restriction have yet to be worked out, and we therefore allow non-canonical scales in this paper.

[14] '+' and '-' are two-place operators that concatenate degree intervals. Since the concatenation of degree intervals is not commutative (see also endnote 10), Bierwisch (1984) distinguishes between the *internal* and the *external* argument of '+' or '-', y and x respectively in $[x[+y]]$.

our $dim_{<D,x>}$. $\supset$ and $\subset$ are containment relations on degree intervals, i.e. $x \subset y$ means that the degree interval x is contained in y. (21) gives example sentences containing absolute DAs and their representations (**b** stands for *board*):

(21) a. The board is 5m long.
[QUANT MAX $\mathbf{b}$] $\supset$ [0 [$+5m$]]
b. The board is long.
$\exists x_i$ [[QUANT MAX $\mathbf{b}$] $\supset$ [s [$+x_i$]]]
c. The board is short.
$\exists x_i$[[QUANT MAX $\mathbf{b}$] $\subset$ [s [$\Leftrightarrow x_i$]]]

These representations are equivalent to their VSS counterparts. But the way they are obtained differs fundamentally from the VSS analysis. First, note that the denotations of the GAs do not contain an existential quantifier, and no reference to a standard, but a free variable v. We thus have to explain how v receives the value 0 in (21)a. and s in (21)b.,c., and how the existential quantifiers are introduced in (21)b.,c. The latter is of less interest to us here. Suffice it to say that in Bierwisch's general semantic theory any syntactically unrealized argument is replaced with an existential quantifier in semantic form. As (21)b. and c. do not contain an overt degree argument, λc is replaced by $\exists x_i$. In (21)a. on the other hand, the degree argument is overtly realized as the MP $5m$, and it fills the argument position c. Thus, in his account too the incorporation of MPs is compositional in a very straightforward way. What interests us here, however, is how the value for the free variable v in the GA denotations is instantiated as 0 in (21)a. and as s in (21)b. and c. Bierwisch (1984) introduces the so-called v-conditions on semantic constants in (22)a.-c., and the principle of comparison value selection in (22)d.

(22) a. The internal argument of $\supset$ or $\subset$ requires an extent, i.e. the initial part of a scale as its argument.
b. If the internal argument of '+' or '-' consists of a numerical expression, the external argument cannot be s.
c. If the internal argument of '+' is an $\exists$-quantified variable, the external argument cannot be 0.
d. The variable v assumes, preferentially, the value 0. If this produces a conflict with one of the previous conditions, v assumes the value s. 0 and s are the only possible values for v

These conditions define the values of v in (21): (21)a. does not violate any conditions and therefore has to be 0 according to (22)d. Condition

(22)c. rules out 0 for (21)b., since the argument of + is ∃-quantified, and (22)a. rules out 0 for (21)c., since $0 \Leftrightarrow c$ is not a scale segment. The conditions furthermore explain the unacceptability of

(23) # The board is 5m short.
$[\textsc{quant max } \mathbf{b}] \subset [v\ [\Leftrightarrow 5m]]$

Here, v cannot be 0, because $0\ [\Leftrightarrow 5m]$ is not an initial scale segment, and because $5m$ is a numerical expression, (22)b. thus rules out s as value for v.

This account of the permissibility of MPs with absolute DAs differs from the analysis sketched in section 2.3. The v-conditions are not part of the compositional apparatus, but operate on semantic constants. An analysis that can account for the data within the compositional system is more elegant, and more importantly allows for a uniform analysis of MP modification across different semantic domains.

3.2 Klein's delineation approach

Delineation approaches analyze gradable adjectives such as *tall* relative to a *comparison class* or *context*, such that the adjective divides the comparison class "into the definitely tall things (if there are any), the definitely not-tall things (if there are any) and those things which are neither definitely tall nor definitely not-tall" (von Stechow, 1984). An absolute GA is thus a simple predicate.

Comparative GAs such as *taller* denote relations between two individuals, where the relevant comparison class minimally contains the two compared individuals. Thus, (1)a. is true iff *Sam* is tall relative to the set {*Sam, Jo*} and *Jo* is not tall relative to this set. Delineation analyses have been proposed e.g. by Lewis (1970); Klein (1980, 1991); Wheeler (1972). While this approach is more attractive than the degree-based approaches for its simplicity, and because it captures the intuition that simple comparatives are statements about two individuals and not degrees, the more complex machinery of a degree ontology has been argued to be necessary to account for phenomena that cannot be handled by delineation approaches, e.g. the analysis of MPs (see von Stechow (1984) for discussion). Klein has responded to this criticism (Klein, 1991); he sketches a different version of a delineation approach that can handle MPs. This account will be discussed below. Delineation approaches will also have difficulties accounting for the converse relation that holds between the comparatives of DA antonyms (see the paragraph on Bierwisch (1984) in section 2.1). This problem will not be discussed here.

Klein (1991) presents a delineation account that does not use comparison classes as described above, but analyzes a DA such as *tall* as *tall(*$\mathbf{s}$*,x)*, where the standard $\mathbf{s}$ determines a delineation [$\mathbf{s}$) relative to a dimension according to which x is judged tall.

A comparative such as *Sam is taller than Jo* is interpreted as (24)

(24) $\exists\mathbf{s}[tall(\mathbf{s}, Sam) \wedge \neg tall(\mathbf{s}, Jo)]$

(24) is true iff there exists a standard according to which Sam is tall, and Jo is not.

In order to incorporate MPs, Klein (1991) uses an equivalent representation to (24) which λ-abstracts over standards. For example, *Sam is taller than Jo* can also be represented as:

(25) $V(\lambda\mathbf{s}[tall(\mathbf{s}, Sam) \wedge \neg tall(\mathbf{s}, Jo)])$

where V is a second order predicate that applies to sets. V is true of a set if the set is non-empty. Thus, (25) states that the set of standards $\mathbf{s}$ according to which Sam is tall, but Jo is not, is non-empty.

For example, if Sam is exactly *1.06m* tall, and Jo *1m*, then $\lambda\mathbf{s}[tall(\mathbf{s}, Sam) \wedge \neg tall(\mathbf{s}, Jo)] = \{$*106, 105, 104, 103, 102, 101*$\}$.

Klein maintains that we can associate a further standard with this set of standards, namely $6cm$. Thus, a comparative containing an MP like *Sue is 6cm taller than Tom* is just a special case of (25) where the set of standards separating Sam from Jo is claimed not just to be nonempty, but equal to $6cm$. While this is fairly intuitive, Klein (1991) does not explain how exactly this association is achieved, but only sketches a possible solution: "If P is a predicate of standards, then the higher order predicate $1m*$ is true of P iff P is true of $1m$ and for any $s > 1m, P$ is false of s. Thus, *Sue is 1m tall* is analyzed as $1m*(\lambda\mathbf{s}[tall(\mathbf{s}, Sue)])$." (Klein, 1991).

While this might work for absolutes, it is not clear how comparatives with MPs would be analyzed. For example, *Sam is taller than Jo* would in this system be represented as $6cm*(\lambda\mathbf{s}[tall(\mathbf{s}, Sue) \wedge \neg tall(\mathbf{s}, Tom)])$, which would be true iff $6cm$ is in the set $\lambda\mathbf{s}[tall(\mathbf{s}, Sue) \wedge \neg tall(\mathbf{s}, Tom)])$ and any $\mathbf{s} > 6cm$ is not, which is obviously not what we want.

The truth conditions for MPs can probably be made to work for both absolutes and comparatives, e.g. by requiring that $6cm*$ is true of P iff the difference between the highest and the lowest standard in P equals $6cm$. But it is hard to see how the MP can be incorporated compositionally. Presumably, the V predicate in (25) is introduced by the DA itself or the comparative morphemes, but the MP would have to apply to the set of standards directly.

Note also that we have now lost (at least) one of the attractive features of the delineation approach, namely that it captured the intuition that a comparative such as *Sam is taller than Jo* expresses a simple relation between two individuals. In Klein's (1991) latest formulation, a comparative is a statement about properties of standards.

4 The Coordination Problem

In this section we will briefly discuss the problem of coordinating GAs, which continues to plague degree-based theories of comparison, and to which we can also not yet offer a definite solution.[15]

(26) a. Jo is taller and older than Sam.
b. Jo is 3cm taller and broader than Sam.

The problem for the VSS analysis with sentences like (26) is how to combine the denotations of *taller* and *older*. They both denote sets of vectors, but these sets are completely disjoint because their members are multiples of different unit vectors. Thus, intersection, the usual operation used to analyze conjunction, will yield the empty set.[16]

But let's assume for the sake of argument that there is a way of mapping *taller* and *older* onto sets of vectors that are not entirely disjoint so that their intersection will be non-empty, for example by mapping the two sets onto an abstract scale. Even then we would not be able to derive the correct interpretations for (26). For example, the MP *3cm* in (26)b. would modify the intersection of *taller* and *broader*, and require its members to be *3cm* long. But recall that all difference vectors in the intersection start at the endpoint of a d-vector associated with Sam and end at the endpoint of a d-vector associated with another individual. Since both the vectors in the intersection and the d-vectors for height and broadness would necessarily have to be on the same abstract scale, we would in fact require that both Sam and Jo are as tall as they are broad, which is obviously non-sense. Other degree-based approaches run into similar problems.

Delineation approaches based on comparison classes, however, can handle (26)a. The phrase *taller and older* can simply be analyzed as the intersection of sets of pairs of individuals. But these accounts cannot

[15] The discussion of this problem is based on comments of Yoad Winter (p.c.)

[16] As Yoad Winter observes, sentences of the form *Jo is taller and older than Sam* cannot generally be analyzed as equivalent to *Jo is taller than Sam and Jo it heavier than Sam*, since the equivalence does not hold for all such examples, cf. *Sam is taller and heavier than some student* $\not\Leftrightarrow$ *Sam is taller than some student and Sam is heavier than some student.*

incorporate MPs. How does Klein (1991), which can handle MPs, fare in this respect?

Recall that in Klein (1991), GAs denote relations between standards and individuals. Based on the representation Klein gives for *Sam is taller than Jo* ($V\lambda\mathbf{s}[tall(\mathbf{s}, Sam) \wedge \neg tall(\mathbf{s}, Jo)]$, see section 3.2), we assume that **taller**$'$ denotes the two-place relation $\lambda x.\lambda y.V(\lambda\mathbf{s}[tall(\mathbf{s}, x) \wedge \neg tall(\mathbf{s}, y)])$. Intersection with **broader**$'$ would result in $\lambda x.\lambda y.V(\lambda\mathbf{s}[tall(\mathbf{s}, x) \wedge \neg tall(\mathbf{s}, y)]) \wedge V(\lambda\mathbf{s}[broad(\mathbf{s}, x) \wedge \neg broad(\mathbf{s}, y)])$. Thus, in Klein (1991), too, the phrase *taller and broader* can be analyzed as intersection of sets of pairs of individuals. However, as already pointed out in section 3.2, it is not clear at which point an MP would enter the derivation for a single comparative DA, and it is even less clear how the MP would come to modify both conjuncts in the representation of a coordinated comparative.

Thus, it appears that analyses that can account for coordination have difficulties accounting for MPs and vice versa.

We believe however that this problem can be solved if DAs are allowed to change what kinds of entities they relate at the different compositional stages. In particular we suggest that DA-roots should be analyzed as relations between individuals, and only the addition of the comparative morpheme converts them into relations between individuals and vectors or degrees. Coordination would then take place before the comparative morpheme applies. We are currently working on the VSS realization of such an analysis.

5 Conclusion

We hope to have demonstrated in this paper that simple DAs can straightforwardly be analyzed within VSS and that VSS provides an elegant way to account for MP modification across domains. We have seen in the last two sections that, in general, there appears to be a trade-off between being able to account for MP modification and being able to account for coordination. Further research is required to develop an account that can provide analyses for both phenomena.

VSS is a flexible enough framework to accommodate semantic analyses for different domains, and it is therefore worthwhile to develop VSS analyses for temporal and possibly causal relations as well. We expect a VSS analysis of temporal PPs to be quite similar to that of locative PPs. The analysis we envision for causatives is based on force dynamics as developed by Talmy (1988), in which verbs of obligation and permission are analyzed in terms of interacting forces.

Drawing together analyses for these different empirical phenomena within a single semantic framework should enable us to explain why some languages use the same syntactic constructions to encode all of them and capture the underlying common cognitive basis. In addition to bringing out these similarities, it is desirable that VSS contribute new semantic insights in each of the individual domains. One of VSS' contributions in the analysis of DAs is the ability to account for the distribution of MPs with DAs and the requirement that the standard s be zero in the presence of an MP. Zwarts and Winter (1997) have furthermore proposed a number of language universals concerning locatives based on the monotonicity properties of sets of vectors. We expect that the study of the other empirical phenomena using the tools provided by VSS will lead to the discovery of more language universals of this kind. More concretely with respect to comparatives, future work should extend the analysis to evaluative adjectives such as *beautiful, intelligent* and investigate the relation of adjectival comparatives and generalized quantifiers of the form *more A than B C*, e.g. *More whales than elephants swim.*

References

Bierwisch, M. 1984. The semantics of gradation. In Bierwisch, M. and E. Lang, editors, *Dimensional Adjectives*, pages 71–262. Springer Verlag.

Cresswell, M. J. 1976. The semantics of degree. In Partee, B. et al., editors, *Montague Grammar*, pages 261–292. Academic Press.

Faller, M. 1998a. A vector space semantics for comparatives. Unpublished ms.

Faller, M. 1998b. A vector space semantics for dimensional adjectives. In *ESSLLI'98 Student Session Proceedings.*

Hellan, L. 1981. *Towards an Integrated Analysis of Comparatives.* Gunter Narr Verlag.

Kennedy, C. 1997. *Projecting the Adjective: The Syntax and Semantics of Gradability and Comparison.* PhD thesis, University of California, Santa Cruz.

Klein, E. 1980. A semantics for positive and comparative adjectives. *Linguistics and Philosophy*, 4:1–45.

Klein, E. 1991. Comparatives. In von Stechow, A. and D. Wunderlich, editors, *Semantik: Ein internationales Handbuch der zeitgenössischen Forschung.* Walter de Gruyter.

Kowalski, H.-J. 1967. *Lineare Algebra.* Walter de Gruyter.

Lang, S. 1971. *Lineare Algebra*. Addison-Wesley.

Lewis, D. K. 1970. General semantics. *Synthese*, 22:18–67.

Mey, J. 1976. Comparatives in eskimo. In Hamp, E. P., editor, *Papers on Eskimo and Aleut Linguistics*, pages 159–178. Chicago Linguistic Society.

Rusiecki, J. 1984. *Adjectives and Comparison in English*. Longman.

Seuren, P. 1973. In Kiefer, F. and N. Ruwet, editors, *Generative Grammar in Europe*, pages 528–564. Reidel.

Stassen, L. 1984. The comparative compared. *Journal of Semantics: Special Issue on the Comparative Construction*, 3(1/2):143–182.

von Stechow, A. 1984. Comparing semantic theories of comparison. *Journal of Semantics: Special Issue on the Comparative Construction*, 3(1/2).

Talmy, L. 1988. Force dynamics in language and cognition. *Cognitive Science*, 12(1):49–100.

Wheeler, S.-C. 1972. Attributives and their modifiers. *Nous*, 6(4):310–334.

Winter, Y. 1999. Modification of prepositions and degree adjectives in vector space semantics: Extending the analogy. Unpublished ms.

Zwarts, J. 1997. Vectors as relative positions: A compositional semantics of modified PPs. *Journal of Semantics*, 14:57–86.

Zwarts, J. and Y. Winter. 1997. A semantic characterization of locative PPs. *Journal of Logic, Language and Information*. To appear.