Consider the successive powers of the adjacency matrix A of a directed graph $G(V, E)$. That is, consider the sequence of matrices A^k defined by $A^k = AA^{k-1}$ and $A^1 = A$.

Which, if any, of the following statements are true?

(a) If, for some positive integer k_0, the matrix A^{k_0} has strictly positive entries, then G is strongly connected.

(b) If, for some positive integer k_0, the matrix A^{k_0} has strictly positive entries, then A^k has strictly positive entries for all $k \geq k_0$.

(c) If G is strongly connected, then there exists a positive integer $k_0 > 0$ such that A^{k_0} has strictly positive entries.

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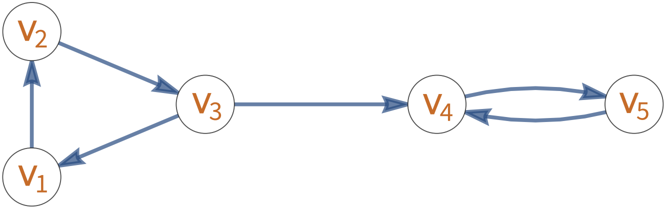
What is the value of $A^{31}_{3,4}$ when A is the adjacency matrix of the graph below? Explain carefully how you obtain your answer.



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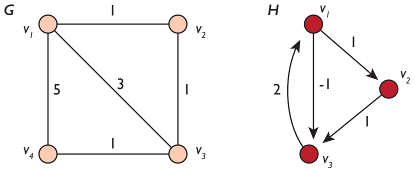
Distance and Tropical Matrix Powers



For each of the graphs above:

(a) By inspection, construct a matrix D whose entries are

$$D_{j,k} = \begin{cases} d(v_j, v_k) & \text{if } v_k \text{ is reachable from } v_j \\ \infty & \text{otherwise} \end{cases}$$

Here $d(v_j, v_k)$ is the weight of a minimal-weight walk from v_j to v_k.

(b) Construct the weight matrix W whose entires are

$$W_{j,k} = \begin{cases} 0 & \text{if } j = k \\ w(v_j, v_k) & \text{if } j \neq k \text{ and } (v_j, v_k) \in E \\ \infty & \text{otherwise} \end{cases}$$

(c) Finally, compute the tropical matrix power $W^{\otimes(n-1)}$ where $n = |V|$ is the number of vertices in the graph. The results should agree with your answers to part (a).

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