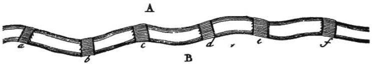

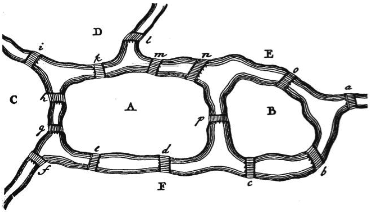

The drawings above come from Euler's original paper on the Königsberg Bridge Problem. For each of the maps above:

- Draw a graph that illustrates the way the bridges connect the land masses.
- Determine whether the graph is Eulerian and, if it isn't, find the smallest number of bridges that you can add to the map so that the corresponding graph becomes Eulerian.

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Recall our three main theorems about Hamiltonian graphs:

Theorem (Dirac, 1952)

Let $G(V, E)$ be a graph with $n \geq 3$ vertices. If $\deg(v) \geq n/2$ for all $v \in V$, then G is Hamiltonian.

Theorem (Ore, 1960)

Let G be a graph with $n \geq 3$ vertices. If

$$\deg(u) + \deg(v) \geq n$$

for every pair of non-adjacent vertices u and v, then G is Hamiltonian.

Theorem (Bondy and Chvátal, 1976)

A graph G is Hamiltonian if and only if its closure $[G]$ is Hamiltonian.

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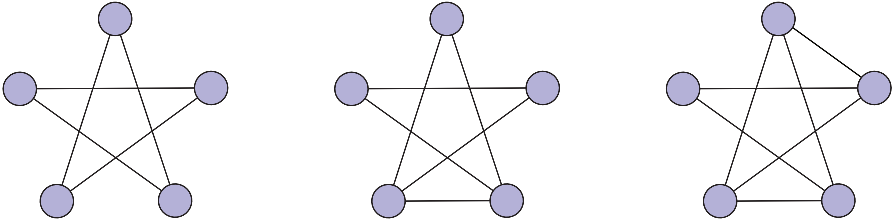

1. Which, if any, of these graphs is Hamiltonian?
2. Construct the closures of these graphs.
3. Which, if any, of these graphs can be proven to be Hamiltonian using:
    - Dirac's Theorem?
    - Ore's Theorem?
    - The Bondy-Chvátal Theorem?

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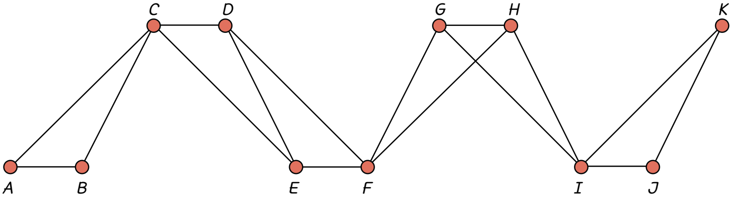

- Find a Hamiltonian *path* in the graph above.
- If you remove the vertices B, E and H, how many connected components remain?
- If you remove the vertices C, F and I, how many connected components remain?

In general, if $G(V, E)$ is a graph that contains a Hamiltonian path and you remove k vertices, what are the largest and smallest numbers of connected components that can remain?

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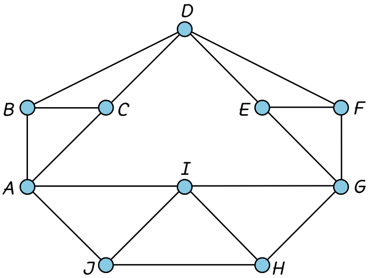

- Find a Hamiltonian *tour* in the graph above.
- If you remove the vertices C, G and J, how many connected components remain?
- If you remove the vertices A, D and G, how many connected components remain?

In general, if $G(V, E)$ is a graph that contains a Hamiltonian cycle and you remove k vertices, what are the largest and smallest numbers of connected components that can remain?

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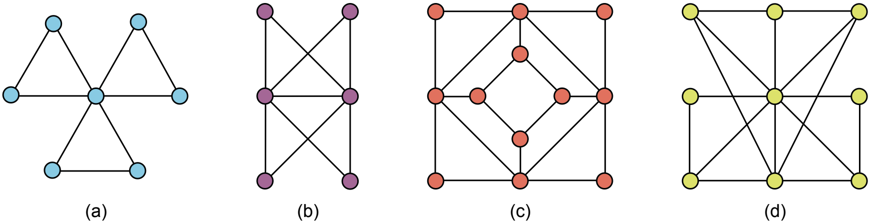
Lemma (Subgraph test for a Hamiltonian path)

If a graph $G(V, E)$ contains a set of $k > 0$ vertices such that, when these vertices are removed, the resulting subgraph has more than $k + 1$ connected components, then G cannot contain a Hamiltonian path.

Lemma (Subgraph test for a Hamiltonian tour)

If a graph $G(V, E)$ contains a set of $k > 0$ vertices such that, when these vertices are removed, the resulting subgraph has more than k connected components, then G cannot contain a Hamiltonian tour.

These slides are available on Blackboard and at https://bit.ly/3vPKWsL



- Which, if any, of the graphs above can't contain a Hamiltonian path?
- Which, if any, of the graphs above can't contain a Hamiltonian tour?

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