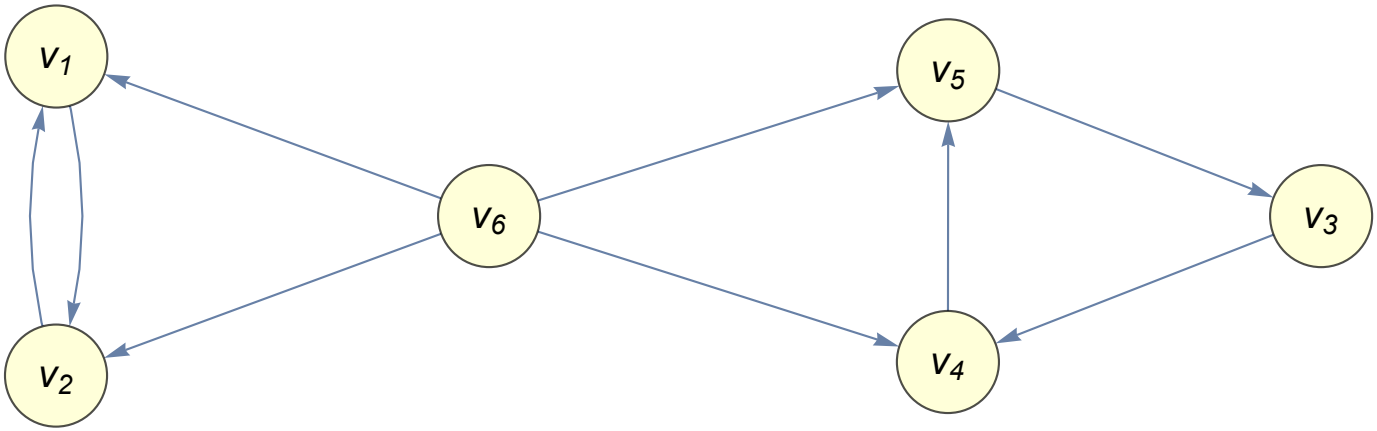
Spregs and cycles



(a) How many spregs with distinguished vertex v_6 appear the graph above? Draw them all.

(b) For each one, list the cycles, if any, that it contains.

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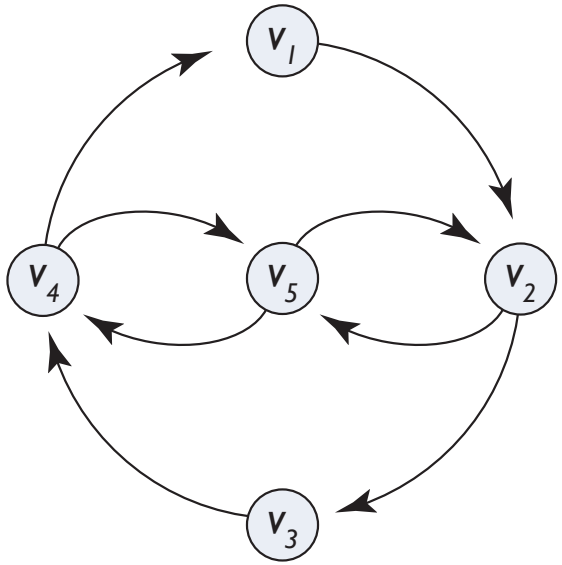
After Problem A2 from 2017's exam



(a) How many spregs with distinguished vertex v_1 appear the graph above? Draw them all.

(b) How many spanning arborescences rooted at v_1 are contained in the graph above? Answer this in two ways: by counting graphs spregs drawn in the previous step and by using the Tutte's Matrix-Tree Theorem.

(c) How many cycles does the graph above contain? For each, determine how many spregs with distinguished vertex v_1 contain them.

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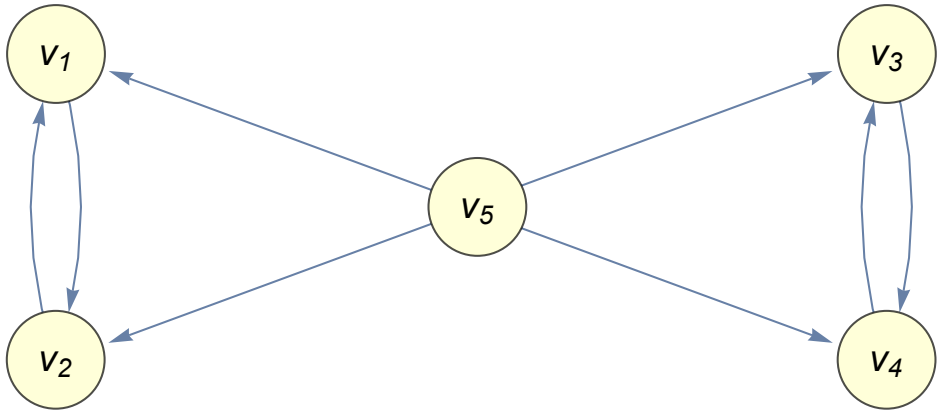
Similar to a quiz problem for Video 6.4



Construct the Laplacian L of the graph above, then fill in the table below, which is about terms in $\det(\mathcal{L})$, where $\mathcal{L} = \hat{L}_5$ is the matrix formed by deleting the 5th row and column of L.

σ	Term in $\det(\mathcal{L})$ $\operatorname{sgn}(\sigma)\prod_{j=1}^{4}\mathcal{L}_{j,\sigma(j)}$	Value of term	Spregs counted
id	$1 \times \mathcal{L}_{1,1}\mathcal{L}_{2,2}\mathcal{L}_{3,3}\mathcal{L}_{4,4}$	16	all spregs
	$-1 \times \mathcal{L}_{1,2}\mathcal{L}_{2,1}\mathcal{L}_{3,3}\mathcal{L}_{4,4}$	- 4	
$(3,4)$			
$(1,2,3)$			
	$1 \times \mathcal{L}_{1,2}\mathcal{L}_{2,1}\mathcal{L}_{3,4}\mathcal{L}_{4,3}$		

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