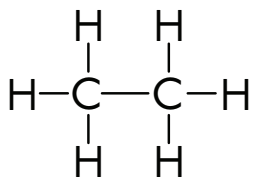
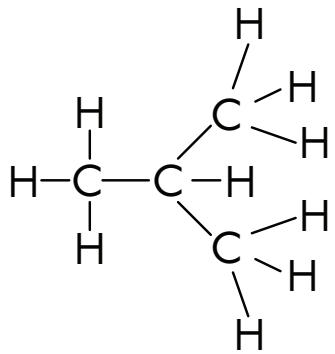
From 2018's exam

A *saturated hydrocarbon* is a molecule with chemical formula C_mH_n (that is, it contains m atoms of carbon and n of hydrogen) in which, as in the examples below, every carbon atom C has four bonds; every hydrogen atom H has one bond; there are no double, triple or quadruple bonds and no sequence of bonds forms a cycle.





Prove the following results:

- If there is a saturated hydrocarbon with formula C_mH_n, then $n = 2m + 2$. *Hint: think about the Handshaking Lemma*
- If m is a positive integer and $n = 2m + 2$, then there exists an example of a saturated hydrocarbon with formula C_mH_n.

These slides are available at https://bit.ly/2O4hhLb

Chromatic Number of Trees

Prove the following statement or find a counterexample:

- If $G(V, E)$ is a tree with $|V| \geq 2$, then $\chi(G) = 2$.

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Draw four non-isomorphic trees on 8 vertices and, for each one, do the following:

- find a 2-colouring ϕ;
- define two subsets of the vertex set, $V_1 \subseteq V$ and $V_2 \subseteq V$, by

$$V_j = \{v \in V \mid \phi(v) = j\};$$

- Compute

$$\sum_{v \in V_1} \deg(v) \qquad \text{and} \qquad \sum_{u \in V_2} \deg(u).$$

Conjecture and prove a lemma based on your observations.

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Degrees when $\chi(G) = 2$

Is it possible for a graph $G(V, E)$ to have $|V| = 17$, $|E| = 73$ and $\chi(G) = 2$?
If so, give an example, but if not, explain rigorously why it is impossible.

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