The rate of growth of a function $f : \mathbb{N} \to \mathbb{R}^+$ is often characterised in terms of some simpler function $g : \mathbb{N} \to \mathbb{R}^+$ in the following ways:

- $f(n) = O(g(n))$ if $\exists\, c_1 > 0$ such that, for all sufficiently large n, $f(n) \leq c_1\, g(n)$;
- $f(n) = \Omega(g(n))$ if $\exists\, c_2 > 0$ such that, for all sufficiently large n, $f(n) \geq c_2\, g(n)$;
- $f(n) = \Theta(g(n))$ if $f(n) = O(g(n))$ and $f(n) = \Omega(g(n))$.

Prove the following

(a) if $f_a(n) = n + \ln(n)$ then $f_a(n) = \Theta(n)$.

(b) if $f_b(n) = \lfloor n \rfloor + \lceil n \rceil$ then $f_b(n) = \Theta(n)$.

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(a) Suppose that $G(V, E)$ is a graph with $|V| = 11$ and that $\deg(v) \geq 5$ for all $v \in V$. Prove that G must be connected. *Hint: what can you say about the size of the connected components?*

(b) Suppose that $G(V, E)$ is a graph with $|V| = 15$ and that $\deg v \geq 3$ for all $v \in V$. What is the largest number of connected components that G could have? Draw an example of such a graph.

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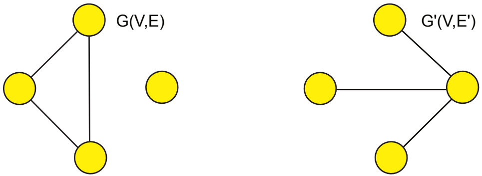
Recall from the review session in Week 1 (or learn for the first time) that the *complement* of a graph $G(V, E)$ is the graph $\overline{G}(V, E')$ whose edge set contains every edge (except loops) that *isn't* in E. That is

$$E' = \{(u, v) \mid u, v \in V,\ u \neq v \text{ and } (u, v) \notin E\}.$$



Prove that if $G_1(V, E_1)$ and $G_2(V, E_2)$ are each other's complements, then one of them must be connected. Is it possible for both to be connected?

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