

Let P_1 and P_2 be two paths of maximum length in a connected graph $G(V, E)$. Prove that P_1 and P_2 have a common vertex.

These slides are available at <https://bit.ly/3yN5N4d>

¹This exercise comes from J. M. Harris, J. L. Hirst, and M. J. Mossinghoff (2008), *Combinatorics and Graph Theory*, Undergraduate Texts in Mathematics, Springer New York, 2nd edition.

Let $T(V, E)$ be a tree with $|V| \geq 2$. Prove that T is bipartite.

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Prove that if every edge of a connected graph $G(V, E)$ lies on an odd number of cycles, then G is Eulerian.

Hint: I found following result, from Video 7.1, useful:

Theorem

Let G be a connected multigraph. Then the following statements are equivalent:

- (a) *G is Eulerian.*
- (b) *Each vertex of G has even degree.*
- (c) *The edge set of G can be partitioned into cycles.*

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