Definition

A *subdivision* of a graph $G(V, E)$ is a graph $H(V', E')$ formed by (perhaps repeatedly) removing an edge $e = (a, b) \in E$ from G and replacing it with a path

$$\{(a, v_1),\ (v_1, v_2),\ \ldots, (v_k, b)\}$$

containing of some number $k \geq 0$ of new vertices $\{v_1, \ldots,\ v_k\}$, each of which has degree 2.

A book[1] that I often recommend offers the following exercises:

(a) Find an example of a connected graph with degree sequence

$$(2, \ldots, 2, 3, 3, 3, 3, 3, 3)$$

that is not a subdivision of $K_{3,3}$.

(b) Find an example of a connected graph with degree sequence

$$(2, \ldots, 2, 4, 4, 4, 4, 4)$$

that is not a subdivision of K_5.

[1] D. A. Marcus (2008), *Graph Theory: A Problem Oriented Approach*, MAA Textbooks, MAA, 2008.

In last week's tutorial we used the bound $m \leq g(n-2)/(g-2)$ to prove that the cube graph I_4 is nonplanar. Kuratowski's theorem then tells us that I_4 must contain a subgraph homeomorphic to $K_{3,3}$ or K_5 (or perhaps both). Find such subgraphs.

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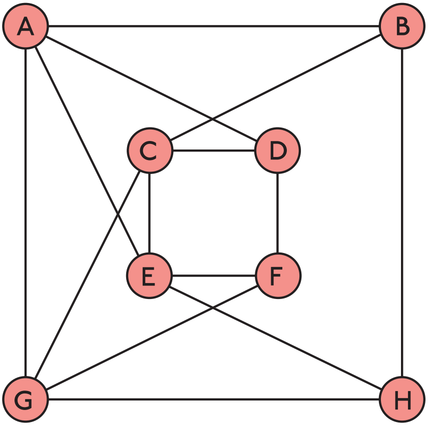
Answer the following questions to prove that the graph at right is non-planar using Kuratowski's Theorem:

(a) Could this graph contain a subgraph homeomorphic to K_5?
(b) Could this graph contain a subgraph homeomorphic to $K_{3,3}$?
(c) How many edges and vertices would one have to delete?



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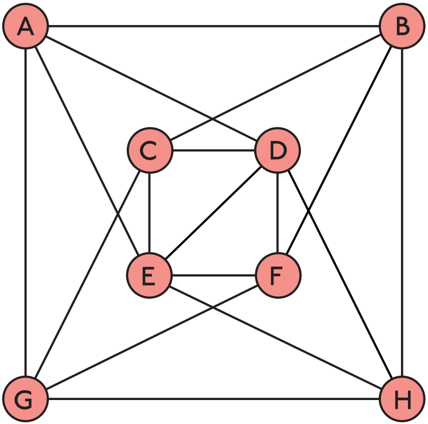
Answer the following questions to prove that the graph at right is non-planar using Kuratowski's Theorem:

(a) Does this graph contain a subgraph homeomorphic to $K_{3,3}$?

(b) Does this graph contain a subgraph homeomorphic to K_5?



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