

We have two theorems that say, roughly, that a planar graph has few edges.

### Theorem (Jungnickel's 1.5.3)

If  $G(V, E)$  is a connected planar graph with  $n = |V|$  vertices and  $m = |E|$  edges then either:

- (A)  $G$  is acyclic and  $m = n - 1$ ;
- (B)  $G$  has at least one cycle and so has a well-defined girth  $g$ . Then  $m \leq g(n - 2)/(g - 2)$ .

### Corollary

If  $G(V, E)$  is a connected planar graph with  $n = |V| \geq 3$  vertices and  $m = |E|$  edges then  $m \leq 3n - 6$ .

Apply them to decide . . .

- Which complete graphs may be planar.
- Which complete bipartite graphs could be planar.
- Which cube graphs could be planar.

These slides are available at <https://bit.ly/3aGhpZJ>