An undirected graph H has adjacency matrix

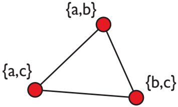
$$A = \begin{pmatrix} 0 & 1 & 1 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 0 & 0 \end{pmatrix}.$$

(i) Without drawing a diagram of H, compute the degrees of all its vertices.

(ii) Now draw H and check your results.

These slides are available at `https://bit.ly/3rx8srF`

The *triangular graph* T_N has vertices labelled by the two-element subsets of a set with N elements. Thus, for example, we could start with a set having three elements—say, $\{a, b, c\}$—then list all of its two-element subsets and regard the result as the vertex set of T_3: $V = \{\{a, b\}, \{a, c\}, \{b, c\}\}$. Pairs of these vertices are adjacent (they have an edge between them) if the subsets that label them have a nonempty intersection, so the corresponding graph looks like the picture below:



Draw diagrams for T_4 and T_5, then show the following:

(i) T_N has $N(N-1)/2$ vertices. *Hint: How many two-elements subsets does a set with N elements have?*

(ii) Each vertex of T_N has degree $2N - 4$.

(iii) If two vertices x and y are adjacent to each other in T_N, then there are $N - 2$ vertices that are adjacent to both.

(iv) If two vertices x and y are *not* adjacent to each other in T_N, then there are 4 vertices that are adjacent to both.