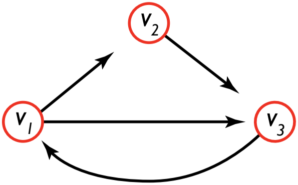
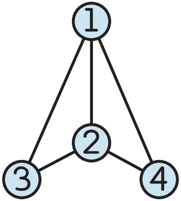
Consider the adjacency matrix A of the graph below and determine whether there are positive integers k such that A^k has exclusively positive entires. If so, find the least such k.



These slides are available on Blackboard and at `https://bit.ly/39cddAp`

Counting Triangles: From B5 in 2022's exam

This problem is adapted from Harris, Hurst & Mossinghof's book *Graph Theory and Combinatorics*.



A *triangle* in an undirected graph is a subgraph isomorphic to the cycle graph C_3.

- How many triangles does the undirected graph above contain?
- Prove that is A is the adjacency matrix of a graph $G(V, E)$, then $\frac{1}{2} A^3_{1,1}$ is the number of triangles in G that include v_1.

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The matrix W below tabulates the edge-weights for a certain weighted, directed graph H where W_{ij} is the weight of the edge (v_i, v_j) and an infinite weight means that the corresponding edge is absent.

$$W = \begin{bmatrix} 0 & 1 & -1 \\ 3 & 0 & 3 \\ \infty & 2 & 0 \end{bmatrix}$$

(a) Draw a diagram of H that includes the edge-weights.

(b) Explain what is meant by the operations $a \oplus b$ and $a \otimes b$ in min-plus algebra.

(c) Recall that the min-plus powers of an $n \times n$ tropical matrix W are defined by

$$W^{\otimes(k+1)} = W^{\otimes k} \otimes W \qquad \text{and} \qquad W^{\otimes 1} = W,$$

then compute the least entry in $W^{\otimes 5}$, where W is the weight matrix above.

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