

Prove the following statement or give a counterexample:

- If two trees have the same degree sequence, then they are isomorphic.

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Suppose that $G(V, E)$ is a graph with $|V| = 5$ vertices and $|E| = 7$ edges and no loops: list all the degree sequences that G could have.

- (a) What is the largest number of edges that a graph on 5 vertices can have?
- (b) What is the degree sequence of K_5 ?
- (c) If we remove an edge from K_5 , what degree sequences can the resulting graph have?
- (d) What if we remove two edges? Three?

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Which of the following could be the degree sequence of a graph?

- (a) (1, 1, 2, 2, 2, 3, 3)
- (b) (3, 3, 4, 4, 4, 5, 5, 5)
- (c) (3, 3, 4, 4, 4, 5, 5)
- (d) (1, 2, 3, 4, 5, 6, 7)
- (e) (4, 4, 4, 5, 5, 5, 5)
- (f) (0, 2, 2, 3, 4, 5, 6)

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This is *not examinable*, but it's interesting and provides an easy approach to problems about degree sequences. There is an algorithm that allows one to decide whether a given sequence \mathcal{D} can be the degree sequence of a graph. It depends on the following result

Theorem (V. Havel (1955) and S.L. Hakimi (1962))

Given a sequence $\mathcal{D} = (d_1, d_2, \dots, d_n)$ with $d_j \leq d_{j+1}$, there is a graph having \mathcal{D} as its degree sequence if and only if there is a graph whose degree sequence is given by $\mathcal{D}' = (d'_1, d'_2, \dots, d'_{n-1})$ with $d'_j \leq d'_{j+1}$, where \mathcal{D}' is formed by:

- removing a largest element, d_n , from \mathcal{D} ;
- subtracting 1 from each of the d_n largest remaining elements;
- rearranging the result in ascending order (if need be).

Think about how to use this result to decide whether a given sequence can be the degree sequence of a graph, then determine which of the following could be degree sequences:

- (2, 3, 3, 3, 3)
- (2, 2, 4, 4, 4)
- (2, 2, 2, 5, 5, 5, 5)
- (3, 3, 4, 5, 5, 5, 5)

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