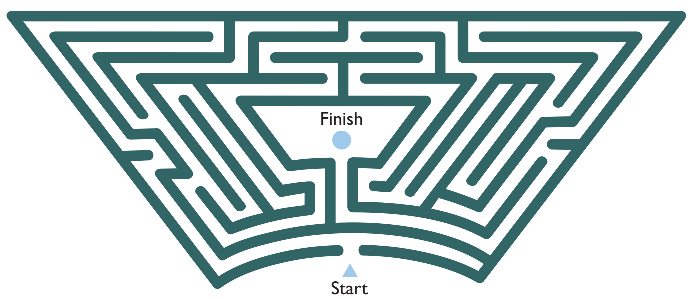
MATH20902: Discrete Maths, Problem Set 1

The problems below are arranged, roughly, in increasing order of difficulty.

- There are only a few problems for Week 1, as we have only just started, but there will be a larger range in later weeks.
- Some of the problems below refer to Dieter Jungnickel's book, *Graphs, Networks and Algorithms* while others come from Harris, Hirst and Mossinghoff's, *Graph Theory and Combinatorics*. PDF versions of both these books are available from within the university's network at https://bit.ly/Jungnickel4 and https://bit.ly/1cp43xp, respectively.

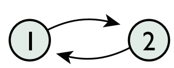
---

(1). The sketch below shows one of the historical configurations of the celebrated hedge maze at Hampton Court Palace in London.



Draw a graph that has vertices at the maze's start and finish (here marked with a triangle and a circle, respectively) as well as vertices at each junction or dead-end. Include edges when two vertices are adjacent as one walks through the maze. Now study the graph and solve the maze.

(2). Construct the adjacency matrix for the graph below.



(3). Make a drawing of a graph whose vertex set is

$$V = \{p,\ q,\ r,\ s,\ t,\ u,\ v,\ w,\ x,\ y,\ z\}$$

and whose adjacency lists are

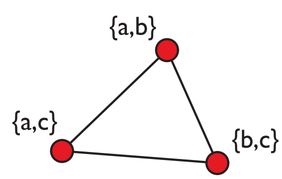
$$\begin{array}{lll} A_p = \{t, u, x\} & A_q = \{y\} & A_r = \{s, u, w\} \\ A_s = \{r, w\} & A_t = \{p, v\} & A_u = \{p, r, w\} \\ A_v = \{t, x, z\} & A_w = \{u, r, s\} & A_x = \{p, v\} \\ A_y = \{q\} & A_z = \{v\} & \end{array}$$

(4). An undirected graph H has adjacency matrix

$$A = \begin{pmatrix} 0 & 1 & 1 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 0 & 0 \end{pmatrix}.$$

Without drawing a diagram of H, compute the degrees of all its vertices.

(5) (Adapted from Jungnickel's Exercise 1.1.3)**.** The *triangular graph* T_N has vertices labelled by the two-element subsets of a set with N elements. Thus, for example, we could start with a set having three elements—say, $\{a, b, c\}$—then list all of its two-element subsets and regard the result as the vertex set of T_3: $V = \{\{a, b\}, \{a, c\}, \{b, c\}\}$. Pairs of these vertices are adjacent (they have an edge between them) if the subsets that label them have a nonempty intersection, so the corresponding graph looks like the picture below:



Draw diagrams for T_4 and T_5, then show the following:

(a) T_N has $N(N-1)/2$ vertices. *Hint: How many two-elements subsets does a set with N elements have?*

(b) Each vertex of T_N has degree $2N - 4$.

(c) If two vertices x and y are adjacent to each other in T_N, then there are $N - 2$ vertices that are adjacent to both.

(d) If two vertices x and y are *not* adjacent to each other in T_N, then there are 4 vertices that are adjacent to both.