Solutions to Additional Questions 6 and 7

6 Arguments in words.

Symbolise the following arguments and prove them valid using the rules of deduction

1. Adam was the first man and Eve the first woman. If the bible is wrong then Adam wasn't the first man. If the bible is right then Eve wasn't the first woman. Therefore, Noah built the Arc.

(Is it also valid to conclude that Noah didn't build the arc?)

Let A= Adam was the first man, E = Eve was the first women, B = the Bible is right, N = Noah built the Arc.
The argument becomes $A \wedge E,\ (\neg B) \rightarrow (\neg A)\,,\ B \rightarrow (\neg E)\ \vdash\ N$.

1	$A \wedge E$	A
2	A	$\wedge$E 1
3	$\neg(\neg A)$	DN 2
4	$(\neg B) \rightarrow (\neg A)$	A
5	$\neg(\neg B)$	MTT 3,4
6	B	DN 5
7	$B \rightarrow (\neg E)$	A
8	$\neg E$	MPP 6,7
9	E	$\wedge$E 1
10	$E \vee N$	$\vee$I 9
11	N	DS 8, 10

You can deduce that Noah didn't build the arc by replacing lines 10 and 11 by

10	$E \vee \neg N$	$\vee$I 9
11	$\neg N$	DS 8, 10

This is just the old result that from a contradiction E and $\neg E$ we can deduce anything.

2. Either the fridge is plugged in or I can see the milk. If the fridge door is open and the fridge is plugged in then the fridge light is on. If the fridge light is off then I cannot see the milk. Therefore, if the fridge door is open then the fridge light is on.

Let P = the fridge is Plugged in, M = I can see the Milk, D = the fridge Door is open, L = The fridge Light is on.

The argument is $P \vee M, (D \wedge P) \to L, (\neg L) \to (\neg M) \ \vdash \ D \to L$.

1	D	A(CP)
2	$P \vee M$	A
3	P	$\vee$E 2
4	$D \wedge P$	$\wedge$I 1,3
5	$(D \wedge P) \to L$	A
6	L	MPP 4,5
7	M	$\vee$E 2
8	$\neg(\neg M)$	DN 7
9	$(\neg L) \to (\neg M)$	A
10	$\neg(\neg L)$	MTT 8,9
11	L	DN 10
12	L	$\vee$E 3-11
13	$D \to L$	CP 1-12

3. Annie cycles to work and Steph walks to the shops. If Annie cycles to work then either Cynthia or Ron walk to the shops, If both Cynthia and Steph walk to the shops then Ron also walks to the shops. Therefore, Ron walks to the shops.

Let $A =$ Annie cycles to work, $S =$ Steph walks to the shops, $C =$Cynthia walks to the shops, $R =$ Ron walks to the shops.

The argument becomes $A \wedge S, \ A \to (C \vee R), \ (C \wedge S) \to R \ \vdash \ R$.

1	$A \wedge S$	A
2	A	$\wedge$E 1
3	$A \to (C \vee R)$	A
4	$C \vee R$	MPP 2,3
5	R	$\vee$E 4
6	C	$\vee$E 4
7	S	$\wedge$E 1
8	$C \wedge S$	$\wedge$I, 6,7
9	$(C \wedge S) \to R$	A
10	R	MPP 8,9
11	R	$\vee$E 5-10

4. If $2+3 \neq 5$ then either $2 \times 3 = 7$ or $2 \times 3 = 5$. But $2+3=5$ only if $6=7$. Yet $6 \neq 7$ and $2 \times 3 \neq 7$. Hence $2 \times 3 = 5$.

Let $p : 2+3=5, \ q : 2 \times 3 = 7, \ r : 2 \times 3 = 5, \ s : 6 = 7$

The argument becomes $(\neg p) \to (q \vee r)\,,\ p \to s,\ \ (\neg s) \wedge (\neg q)\ \ \vdash\ r$

1	$(\neg s) \wedge (\neg q)$	A
2	$\neg s$	$\wedge$E 1
3	$p \to s$	A
4	$\neg p$	MTT 2,3
5	$(\neg p) \to (q \vee r)$	S
6	$q \vee r$	MPP 4,5
7	$\neg q$	$\wedge$E 1
8	r	DS 6,7

5. If either Buxton or Macclesfield lie on the A6 then Stockport is not on the A6. If both Stockport and Longsight lie on the A6 then Buxton is on the A6. Therefore, if Stockport lies on the A6 then Longsight is not on the A6.

Let B = Buxton lies on the A6, M = Macclesfield lies on the A6, L = Longsight lies on the A6, S = Stockport lies on the A6.

The argument becomes $(B \vee M) \to (\neg S)\,, (S \wedge L) \to B\ \vdash\ S \to (\neg L)\,.$

Hint: CP followed by RAA

1	$\lceil$		S	A(CP)
2	$\vert$	$\lceil$	$\neg(\neg L)$	A(RAA)
3	$\vert$	$\vert$	L	DN 2
4	$\vert$	$\vert$	$S \wedge L$	$\wedge$I 1,3
5	$\vert$	$\vert$	$(S \wedge L) \to B$	A
6	$\vert$	$\vert$	B	MPP 4,5
7	$\vert$	$\vert$	$B \vee M$	$\vee$I 6
8	$\vert$	$\vert$	$(B \vee M) \to (\neg S)$	A
9	$\vert$	$\vert$	$\neg S$	MPP 7,8
10	$\vert$	$\lfloor$	$S \wedge (\neg S)$	$\wedge$I 1,9
11	$\lfloor$		$\neg L$	RAA 2-10
12			$S \to (\neg L)$	CP 1-11

7 Invalid Arguments.

Do not draw up truth tables for the following arguments but, instead, find truth values for the variables in each case that make all the premises true and conclusion false. In this way show that each of the following arguments is invalid.

1. $A \to B,\ B\ \vdash\ A,$ Answer: A False, B True

2. $C \to D, \ \neg C \ \vdash \ \neg D,$ Answer: D True, C False

3. $E \vee G, \ G \vee H \ \vdash \ E \vee H,$ Answer: E and H False, G True

4. $(I \vee J) \to (K \wedge L), \ K \wedge L \ \vdash \ I \vee J,$
Answer: I and J False, K and L True (Can you see any similarity with Question 1)

5. $(m \vee n) \wedge (m \vee p), \ (n \vee q) \wedge (p \vee q) \ \vdash \ m \vee q,$
Answer: m and q False, n and p True

6. $r \to (s \wedge t), \ s \wedge (\neg u) \ \vdash \ r \leftrightarrow u,$
Answer: s True, u False, r True, t True

7. $v \to (w \vee x), \ y \to (\neg w), \ (\neg z) \vee y, \ x \to (a \wedge (\neg z)), \ z \to a \ \vdash \ b \to (\neg z)$. Answer:

For $b \to (\neg z)$ to be False we need b True, z True.
For $z \to a$ to be True we need a True.
For $x \to (a \wedge (\neg z))$ to be True we need x False.
For $(\neg z) \vee y$ to be True we need y True.
For $y \to (\neg w)$ to be True we need w False.
For $v \to (w \vee x)$ to be True we need v False.

Hence a, b, y and z are TRUE while x, w and v are FALSE