

# Solutions to Additional Questions 1

Those marked with a \* may be considered harder than the rest.

## 1 Valid Arguments

Prove the following arguments are valid:

1.  $(A \wedge B) \vee C, \neg(A \wedge B) \vdash C,$

$$\begin{array}{ll} 1 & (A \wedge B) \vee C \quad \text{A} \\ 2 & \neg(A \wedge B) \quad \text{A} \\ 3 & C \quad \text{DS 1,2} \end{array}$$

2.  $(D \rightarrow E) \rightarrow (F \rightarrow G), D \rightarrow E \vdash F \rightarrow G,$

$$\begin{array}{ll} 1 & (D \rightarrow E) \rightarrow (F \rightarrow G) \quad \text{A} \\ 2 & D \rightarrow E \quad \text{A} \\ 3 & F \rightarrow G \quad \text{MPP 1,2} \end{array}$$

3.  $m \rightarrow (p \vee q), p \rightarrow q, m \vdash q,$

$$\begin{array}{ll} 1 & m \rightarrow (p \vee q) \quad \text{A} \\ 2 & m \quad \text{A} \\ 3 & p \vee q \quad \text{MPP1,2} \\ 4 & [ \quad q \quad \text{A}(\vee E) \\ 5 & [ \quad p \quad \text{A}(\vee E) \\ 6 & | \quad p \rightarrow q \quad \text{A} \\ 7 & | \quad q \quad \text{MPP 1,2} \\ 8 & | \quad q \quad \vee E4 - 7 \end{array}$$

Note how the first subproof starts and finishes on the same line.

4.  $m \rightarrow (p \vee q), p \rightarrow r, \neg r, m \vdash q,$

$$\begin{array}{ll} 1 & m \rightarrow (p \vee q) \quad \text{A} \\ 2 & m \quad \text{A} \\ 3 & p \vee q \quad \text{MPP1,2} \\ 4 & [ \quad q \quad \text{A}(\vee E) \\ 5 & [ \quad p \quad \text{A}(\vee E) \\ 6 & | \quad p \rightarrow r \quad \text{A} \\ 7 & | \quad r \quad \text{MPP 1,2} \\ 8 & | \quad r \vee q \quad \vee I 7 \\ 9 & | \quad \neg r \quad \text{A} \\ 10 & | \quad q \quad \text{DS 8,9} \\ 11 & | \quad q \quad \vee E4 - 7 \end{array}$$

This proof contains, on lines 7-10, the common little result that from a contradiction (here  $r$  and  $\neg r$ ) we can deduce anything (here  $q$ ).

5.  $m \rightarrow (p \vee q), p \rightarrow r, \neg r \vdash m \rightarrow q,$

1	[	$m$	A(CP)
2		$m \rightarrow (p \vee q)$	A
3		$p \vee q$	MPP 1,2
4		[	A( $\vee E$ )
5			A( $\vee E$ )
6			A
7			MPP 1,2
8			$\vee I$ 7
9			A
10			DS 8,9
11			$\vee E$ 4-7
12			CP 1-11

So lines 2-11 are identical to those seen in Question 4.

6. Give a proof using RAA, **not** MPP, of  $q, q \rightarrow s \vdash s,$

1	[	$\neg s$	A(RAA)
2		$q \rightarrow s$	A
3		$\neg q$	MTT 1,2
4		$q$	A
5		[	$\wedge I$ 3,4
6			RAA 1-5

7. Give a proof using RAA, **not** MTT, of  $q \rightarrow s, \neg s \vdash \neg q,$

1	[	$\neg(\neg q)$	A(RAA)
2		$q$	DN 1
3		$q \rightarrow s$	A
4		$s$	MPP 1,2
5		$\neg s$	A
6		[	$\wedge I$ 3,4
7			RAA 1-5

8. Use RAA to prove the following:

8.1.  $(\neg p) \rightarrow t, \neg(s \vee t) \vdash p,$

1	[	$\neg p$	A(RAA)
2		$(\neg p) \rightarrow t$	A
3		$t$	MPP 1,2
4		$s \vee t$	$\vee$ I 3
5		$\neg(s \vee t)$	A
6		$(s \vee t) \wedge (\neg(s \vee t))$	$\wedge$ I 4,5
7		$p$	RAA 1-6

8.2.  $(\neg s) \rightarrow p, \neg(s \vee t) \vdash p,$

1	[	$\neg p$	A(RAA)
2		$(\neg s) \rightarrow p$	A
3		$\neg(\neg s)$	MTT 1,2
4		$s$	DN 3
5		$s \vee t$	$\vee$ I 4
6		$\neg(s \vee t)$	A
7		$(s \vee t) \wedge (\neg(s \vee t))$	$\wedge$ I 5,6
8		$p$	RAA 1-7

8.3  $p \vee s, \neg(s \vee t) \vdash p.$

1	[	$\neg p$	A(RAA)
2		$p \vee s$	A
3		$s$	DS 1,2
4		$s \vee t$	$\vee$ I 3
5		$\neg(s \vee t)$	A
6		$(s \vee t) \wedge (\neg(s \vee t))$	$\wedge$ I 4,5
7		$p$	RAA 1-6

9. Use C.P. to prove  $a \rightarrow (b \vee r), \neg r \vdash a \rightarrow b.$

1	[	$a$	A(CP)
2		$a \rightarrow (b \vee r)$	A
3		$b \vee r$	MPP 1,2
4		$\neg r$	A
5		$b$	DS 3,4
6		$a \rightarrow b$	CP 1-5

10. \* Prove  $a \rightarrow (b \vee r)$ ,  $(a \rightarrow b) \rightarrow c$ ,  $\neg(c \vee r) \vdash r$ . (Try to reuse the proof of Qu. 9)

1	[	$\neg r$	A(RAA)
2		[	$a$ A(CP)
3			$a \rightarrow (b \vee r)$ A
4			$b \vee r$ MPP 2,3
5			$b$ DS 1,4
6			$a \rightarrow b$ CP 2-5
7			$(a \rightarrow b) \rightarrow c$ A
8			$c$ MPP 6,7
9			$c \vee r$ $\vee$ I 8
10			$\neg(c \vee r)$ A
11			$(c \vee r) \wedge (\neg(c \vee r))$ $\wedge$ I 9,10
12			$r$ RAA 1-11

11.  $G \vee H$ ,  $(\neg H) \vee I$ ,  $(\neg G) \vee J \vdash I \vee J$ .

1		$G \vee H$	A
2		[	$G$ $\vee$ E 1
3			$\neg(\neg G)$ DN 2
4			$(\neg G) \vee J$ A
5			$J$ DS 3,4
6			$I \vee J$ $\vee$ I 5
7			$H$ $\vee$ E 1
8			$\neg(\neg H)$ DN 7
9			$(\neg H) \vee I$ A
10			$I$ DS 8,9
11			$I \vee J$ $\vee$ I 10
12			$I \vee J$ $\vee$ E 2-11