MATH10242 Sequences and Series: Exercises 7, for Week 8 Tutorials

Question 1 i. Show that

$$\left(\frac{1}{n^3 - 2n + 2}\right)_{n\geq 1}$$

is a subsequence of $(1/n)_{n\geq 1}$.
ii. Is

$$\left(\frac{2}{6^n}\right)_{n\geq 1}$$

a subsequence of $(1/n)_{n\geq 1}$? Justify your answer.

Question 2 If $(b_n)_{n\geq 1}$ is a subsequence of $(a_n)_{n\geq 1}$ and $(c_n)_{n\geq 1}$ a subsequence of $(b_n)_{n\geq 1}$ show that $(c_n)_{n\geq 1}$ is a subsequence of $(a_n)_{n\geq 1}$.

Question 3: Using L'Hôpital's Rule, or otherwise, find the limit of the sequences

(i) $\left(\dfrac{\ln(7n^{1/4} - 2)}{\ln(n+1)}\right)_{n\in\mathbb{N}}$ (ii) $\left(\dfrac{e^{e^n}}{e^n}\right)_{n\geq 1}$

(iii) $\left(\dfrac{1 - e^{-n}}{2 - e^{-2n}}\right)_{n\geq 1}$ (iv) $\left(\dfrac{1 - e^{n}}{2 - e^{2n}}\right)_{n\geq 1}$

Question 4: (i) Use L'Hôpital's Rule to show that

$$\frac{(\ln n)^2}{n} \to 0$$

as $n \to \infty$.
(ii) Show by induction that for any $k \in \mathbb{N}$,

$$\frac{(\ln n)^k}{n} \to 0$$

as $n \to \infty$.

Question 5: (i) Using the formula

$$(x - y) = \frac{(x-y)(x^2 + xy + y^2)}{(x^2 + xy + y^2)} = \frac{(x^3 - y^3)}{(x^2 + xy + y^2)}$$

or otherwise, find

$$\lim_{n\to\infty} \sqrt[3]{n^3 + n^2} - n.$$

(ii) Show that $[\sqrt[3]{n^3 + n^2}\,] = n$.

(iii) Using subsequences show that $[\sqrt[3]{n}\,] - \sqrt[3]{n}$ does not have a limit.