Problem Sheet for Week 6 | ***MT1121*** | **More Integration**

***Suggested reading*:** 'Stewart' Chapters 5, 6 and 7 (and chapters studied earlier)

Easy Questions

1. Evaluate the following indefinite integrals (you should be able to write down the answers immediately)

(a) $\displaystyle\int \frac{14x^6 - 3x^2}{2x^7 - x^3 + 1}\,\mathrm{d}x$ (b) $\displaystyle\int \frac{\sec^2\theta}{\tan\theta}\,\mathrm{d}\theta$ (c) $\displaystyle\int \frac{1/r}{\ln|r|}\,\mathrm{d}r$

(d) $\displaystyle\int \frac{1/\sqrt{1+t^2}}{\sinh^{-1} t}\,\mathrm{d}t$ (e) $\displaystyle\int \frac{1/\sqrt{1-s^2}}{\sin^{-1} s}\,\mathrm{d}s$ (f) $\displaystyle\int \frac{1/(1+z^2)}{\tan^{-1} z}\,\mathrm{d}z$

(g) $\displaystyle\int \frac{\sin\theta}{\cos\theta}\,\mathrm{d}\theta$ (h) $\displaystyle\int \frac{\cos\theta}{\sin\theta}\,\mathrm{d}\theta$ (i) $\displaystyle\int \frac{\sinh y}{\cosh y}\,\mathrm{d}y$

(j) $\displaystyle\int \frac{\cosh y}{\sinh y}\,\mathrm{d}y$ (k) $\displaystyle\int \frac{\mathrm{cosec}^2 u}{\cot u}\,\mathrm{d}u$ (l) $\displaystyle\int \frac{1/\sqrt{1-v^2}}{\cos^{-1} v}\,\mathrm{d}v$

(m) $\displaystyle\int \frac{\mathrm{sech}^2 w}{\tanh w}\,\mathrm{d}w$ (n) $\displaystyle\int \frac{1/\sqrt{r^2-1}}{\cosh^{-1} r}\,\mathrm{d}r$ (o) $\displaystyle\int \frac{\mathrm{cosech}^2 x}{\coth x}\,\mathrm{d}x$

(p) $\displaystyle\int \frac{1/(u^2-1)}{\tanh^{-1} u}\,\mathrm{d}u$ (q) $\displaystyle\int \frac{1/(u^2-1)}{\coth^{-1} u}\,\mathrm{d}u$ (r) $\displaystyle\int \frac{x^n}{x^{n+1}+a}\,\mathrm{d}x$

Standard Questions

2. Confirm (by differentiating) that the product $\sin x \cos x$ can be integrated in different ways to give the three results

$$\int \sin x\cos x\,\mathrm{d}x = \tfrac{1}{2}\sin^2 x + C,\qquad \int \sin x\cos x\,\mathrm{d}x = -\tfrac{1}{2}\cos^2 x + C,\qquad \int \sin x\cos x\,\mathrm{d}x = -\tfrac{1}{4}\cos(2x) + C$$

Is it true that $\tfrac{1}{2}\sin^2 x = -\tfrac{1}{2}\cos^2 x = -\tfrac{1}{4}\cos(2x)$?

If not, why is it that the integral can be written in any of these three ways?

3. (a) Use integration by parts to evaluate the following integrals

i. $\displaystyle\int \sqrt{4+x^2}\,\mathrm{d}x$ ii. $\displaystyle\int \sqrt{a^2-z^2}\,\mathrm{d}z$ iii. $\displaystyle\int \sqrt{9t^2-3}\,\mathrm{d}t$

(b) Evaluate the same integrals using suitable substitutions.

4. Evaluate the indefinite integrals

(a) $\displaystyle\int x^2\ln(1+x^3)\,\mathrm{d}x$ (b) $\displaystyle\int t\sin(t^2)\,e^{1+2\cos t^2}\,\mathrm{d}t$ (c) $\displaystyle\int \frac{\cos\theta\sin\theta}{2\cos\theta-5}\,\mathrm{d}\theta$

(d) $\displaystyle\int \frac{5-x}{\sqrt{2x-3}}\,\mathrm{d}x$ (e) $\displaystyle\int \cosh(r^3-3r)\,(r^2-1)\,\mathrm{d}r$ (f) $\displaystyle\int z\sqrt{4-z^2}\,\mathrm{d}z$

5. Evaluate the definite integrals

(a) $\displaystyle\int_0^{\frac{1}{2}\sqrt{\pi}} \bigl(x-\tfrac{1}{2}\sqrt{\pi}\bigr)\sin\bigl(\tfrac{1}{2}\pi + x\sqrt{\pi} - x^2\bigr)\,\mathrm{d}x$ (b) $\displaystyle\int_{-2}^{5/2} \frac{t+1}{\sqrt{5-2t}}\,\mathrm{d}t$

(c) $\displaystyle\int_0^{\pi/2} \frac{\sin^2\theta\cos\theta}{2-\sin\theta}\,\mathrm{d}\theta$ (d) $\displaystyle\int_0^3 \frac{4s}{\sqrt[3]{9-s^2}}\,\mathrm{d}s$

6. Evaluate the indefinite integrals

(a) $\displaystyle\int \frac{5x^2-7x+26}{x^2-2x+5}\,\mathrm{d}x$ (b) $\displaystyle\int \frac{12x^2-16x}{4x^2-4x+2}\,\mathrm{d}x$ (c) $\displaystyle\int \frac{18x^3-3x^2+7x-62}{9x^2+12x+20}\,\mathrm{d}x$

(d) $\displaystyle\int \frac{3x^2-6x+6}{(1-2x)(x-1)(x+2)}\,\mathrm{d}x$ (e) $\displaystyle\int \frac{2x^2+21x+35}{(x+3)^2(x+1)}\,\mathrm{d}x$ (f) $\displaystyle\int \frac{16x^2-12x-3}{(2x+3)(4x^2-4x+2)}\,\mathrm{d}x$