Sample Answers for Week 4 $\boxed{MT1121}$ **Derivatives, Series, Complex Numbers**

Easy Questions

1. (a) $y+1=\frac{1}{4}(x-\pi)$ (b) $y+1=-4(x-\pi)$

2. (a) $\dfrac{\mathrm{d}}{\mathrm{d}x}\dfrac{1}{1+x}=-\dfrac{1}{(1+x)^2}$ which is never zero. So there are no critical points.

   (b) $\dfrac{\mathrm{d}}{\mathrm{d}x}(1+x^4)=4x^3=0$ at $x=0$ where $1+x^4=1$.
   The first non-zero derivative at $x=0$ is $\dfrac{\mathrm{d}^4}{\mathrm{d}x^4}(1+x^4)=4!>0$, so the point $(0,1)$ is a minimum.

   (c) $\dfrac{\mathrm{d}}{\mathrm{d}x}(3x+x^3)=3+3x^2$ which is never zero. So there are no critical points.

   (d) $\dfrac{\mathrm{d}}{\mathrm{d}x}(3x-x^3)=3-3x^2=0$ at $x=1$ where $3x-x^3=2$ and $x=-1$ where $3x-x^3=-2$.
   Because these are the only turning points, $(-1,-2)$ is a minimum and $(1,2)$ is a maximum.

   (e) $\dfrac{\mathrm{d}}{\mathrm{d}x}\dfrac{x}{1-x}=\dfrac{(1-x)\times 1-x(-1)}{(1-x)^2}=\dfrac{1}{(1-x)^2}$ which is never zero. So there are no critical points.

   (f) $\dfrac{\mathrm{d}}{\mathrm{d}x}(1-x^{-4})=4x^{-5}$ which is never zero. So there are no critical points.

   (g) $\dfrac{\mathrm{d}}{\mathrm{d}x}x^5=5x^4=0$ at $x=0$ where $x^5=0$.
   The first non-zero derivative at $x=0$ is $\dfrac{\mathrm{d}^5}{\mathrm{d}x^5}x^5=5!$, so $(0,0)$ is a point of inflection.

   (h) $\dfrac{\mathrm{d}}{\mathrm{d}x}(2-x^6)=-6x^5=0$ at $x=0$ where $2-x^6=2$.
   The first non-zero derivative at $x=0$ is $\dfrac{\mathrm{d}^6}{\mathrm{d}x^6}(2-x^6)=-6!<0$, so the point $(0,2)$ is a maximum.

3. (a) $1=e^{i0}$ (b) $-i=e^{i3\pi/2}$

   (c) $i=e^{i\pi/2}$ (d) $-1=e^{i\pi}$

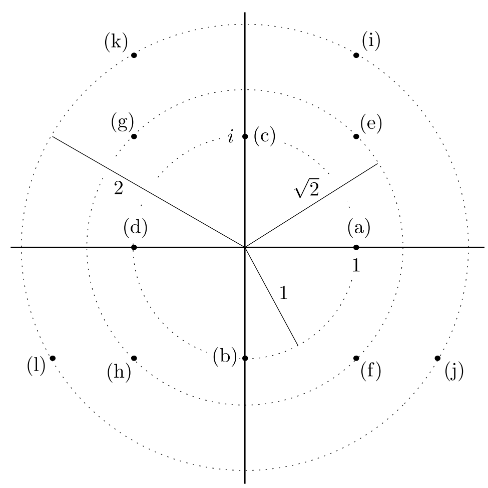
   (e) $1+i=\sqrt{2}e^{i\pi/4}$ (f) $1-i=\sqrt{2}e^{-i\pi/4}$

   (g) $-1+i=\sqrt{2}e^{-i3\pi/4}$ (h) $-1-i=\sqrt{2}e^{-i5\pi/4}$

   (i) $1+i\sqrt{3}=2e^{i\pi/3}$ (j) $\sqrt{3}-i=2e^{i11\pi/6}$

   (k) $-1+i\sqrt{3}=2e^{i2\pi/3}$ (l) $-\sqrt{3}-i=2e^{i7\pi/6}$

   These are not the only answers because the argument can be changed by any multiple of 2π.



4. (a) $$e^x=\sum_{k=0}^{\infty}\frac{x^k}{k!}=1+x+\frac{x^2}{2}+\frac{x^3}{3!}+\frac{x^4}{4!}+\cdots+\frac{x^k}{k!}+\cdots$$

   (b) $$\sin x=\sum_{k=0}^{\infty}(-1)^k\frac{x^{2k+1}}{(2k+1)!}=x-\frac{x^3}{3!}+\frac{x^5}{5!}-\frac{x^7}{7!}+\cdots+(-1)^k\frac{x^{2k+1}}{(2k+1)!}+\cdots$$

   (c) $$\cos x=\sum_{k=0}^{\infty}(-1)^k\frac{x^{2k}}{(2k)!}=1-\frac{x^2}{2}+\frac{x^4}{4!}-\frac{x^6}{6!}+\cdots+(-1)^k\frac{x^{2k}}{(2k)!}+\cdots$$

   (d) $$\ln(1+x)=\sum_{k=1}^{\infty}(-1)^{k+1}\frac{x^k}{k}=x-\frac{x^2}{2}+\frac{x^3}{3}-\frac{x^4}{4}+\cdots+(-1)^{k+1}\frac{x^k}{k}+\cdots$$

5. (a) $i^i = (e^{i\pi/2})^i = e^{-\pi/2}$ ⋆(b) $i^{-i} = (e^{i\pi/2})^{-i} = e^{\pi/2}$

   (c) $(-i)^i = (e^{i3\pi/2})^i = e^{-3\pi/2}$ (d) $\ln i = \ln e^{i\pi/2} = i\pi/2$

   ⋆(e) $\ln(-1) = \ln e^{i\pi} = i\pi$ (f) $\ln(-\sqrt{3}-i) = \ln(2e^{i7\pi/6}) = \ln 2 + i7\pi/6$

   The answers are not unique because the arguments of $i, -i, -1$ and $-\sqrt{3}-i$ can be changed by any multiple of 2π.

⋆6. $\dfrac{8t}{4+t^2}$ for $t \in \mathbb{R}$. The function is zero at $t=0$;

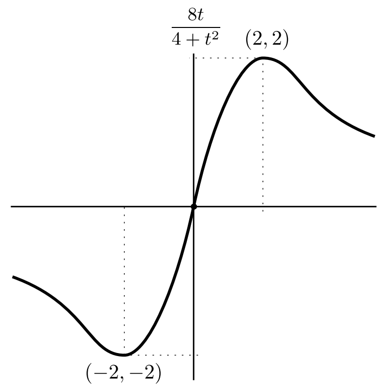
   the function is odd; it approaches zero as $t \to \pm\infty$.

   $$\frac{\mathrm{d}}{\mathrm{d}t}\frac{8t}{4+t^2} = 8\frac{(4+t^2)-t(2t)}{(4+t^2)^2} = 8\frac{4-t^2}{(4+t^2)^2}$$

   which is zero at $t=2$, where $\dfrac{8t}{4+t^2} = \dfrac{16}{4+4} = 2$

   and at $t=-2$, where $\dfrac{8t}{4+t^2} = \dfrac{-16}{4+4} = -2$.

   These are the only critical points and so $(-2,-2)$ is a minimum and $(2,2)$ is a maximum.



7. ⋆(a) The curve A: $2x = \dfrac{y^2}{2} - 2$.

   At $(x,y) = (0,2)$ we have $2{\times}0 = \dfrac{2^2}{2} - 2$ which is true, so $(0,2)$ is on the curve.

   At $\left(-\frac{1}{2}, \sqrt{2}\right)$ we have $2{\times}(-\frac{1}{2}) = \dfrac{(\sqrt{2})^2}{2} - 2 = -1$ which is true, so $\left(-\frac{1}{2}, \sqrt{2}\right)$ is on the curve.

   ⋆(b) Differentiating curve A, gives $2 = y\left.\dfrac{\mathrm{d}y}{\mathrm{d}x}\right|_A$ so that $\left.\dfrac{\mathrm{d}y}{\mathrm{d}x}\right|_A = 2/y$ and $\left.\dfrac{\mathrm{d}y}{\mathrm{d}x}\right|_A = 2/2 = 1$ at $(0,2)$.

   Differentiating curve B $\left(2x = b - \dfrac{y^2}{c}\right)$ gives $2 = -2\dfrac{y}{c}\left.\dfrac{\mathrm{d}y}{\mathrm{d}x}\right|_B$ so that $\left.\dfrac{\mathrm{d}y}{\mathrm{d}x}\right|_B = -c/y$.

   At $(0,2)$ we find $\left.\dfrac{\mathrm{d}y}{\mathrm{d}x}\right|_B = -c/2$ and so for A and B to cross at right angles,

   $\left.\dfrac{\mathrm{d}y}{\mathrm{d}x}\right|_A \left.\dfrac{\mathrm{d}y}{\mathrm{d}x}\right|_B = 1 \times (-c/2) = -1$, so that $c = 2$.

   For curve B to pass through $(0,2)$ we must have $2{\times}0 = b - \dfrac{2^2}{2}$ so that $b = 2$.

   (c) At $\left(-\frac{1}{2}, \sqrt{2}\right)$ we have $\left.\dfrac{\mathrm{d}y}{\mathrm{d}x}\right|_A = 2/y = 2/\sqrt{2} = \sqrt{2}$

   Differentiating curve P $\left(2x = p - \dfrac{y^2}{q}\right)$ gives $2 = -2\dfrac{y}{q}\left.\dfrac{\mathrm{d}y}{\mathrm{d}x}\right|_P$ so that $\left.\dfrac{\mathrm{d}y}{\mathrm{d}x}\right|_P = -q/y$.

   At $\left(-\frac{1}{2}, \sqrt{2}\right)$ we find $\left.\dfrac{\mathrm{d}y}{\mathrm{d}x}\right|_P = -q/\sqrt{2}$, and so for A and P to cross at right angles,

   $\left.\dfrac{\mathrm{d}y}{\mathrm{d}x}\right|_A \left.\dfrac{\mathrm{d}y}{\mathrm{d}x}\right|_P = \sqrt{2} \times (-q/\sqrt{2}) = -1$, so that $q = 1$.

   For curve P to pass through $\left(-\frac{1}{2}, \sqrt{2}\right)$ we must have $2{\times}(-\frac{1}{2}) = p - \dfrac{(\sqrt{2})^2}{1}$ so that $p = 1$.

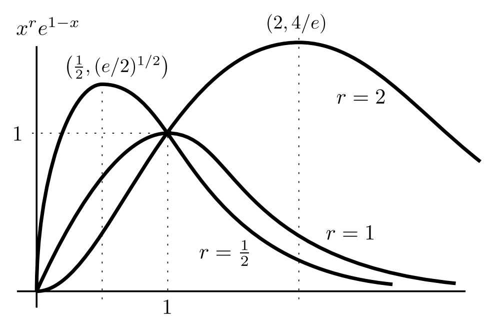
8. (a) For $r = 1$ we consider $y = xe^{1-x}$, giving

   $\dfrac{\mathrm{d}y}{\mathrm{d}x} = e^{1-x} - xe^{1-x} = (1-x)e^{1-x}$

   so that $\dfrac{\mathrm{d}y}{\mathrm{d}x} = 0$ at $x = 1$

   where $y = 1e^{1-1} = 1$.

   This is the only critical point and since $xe^{1-x} \geq 0$ for $x \geq 0$ it must be a maximum.



(b) For $0 < r < 1$ we consider $y = x^r e^{1-x}$, giving
$\frac{dy}{dx} = rx^{r-1}e^{1-x} - x^r e^{1-x} = \frac{r-x}{x^{1-r}} e^{1-x}$
so that $\frac{dy}{dx} = 0$ at $x = r < 1$
where $y = r^r e^{1-r}$.

This is the only critical point and since $x^r e^{1-x} \geq 0$ for $x \geq 0$ it must be a maximum.

Moreover, because the curve passes through $(1,1)$, we must have $r^r e^{1-r} > 1$.

For $r = \frac{1}{2}$ the maximum is at $\left(\frac{1}{2}, (e/2)^{1/2}\right)$.

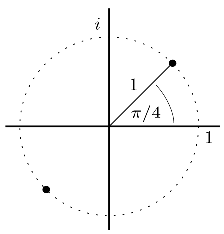
⋆(c) For $r > 1$ we consider $y = x^r e^{1-x}$, giving
$\frac{dy}{dx} = rx^{r-1}e^{1-x} - x^r e^{1-x} = (r-x)x^{r-1}e^{1-x}$
so that $\frac{dy}{dx} = 0$ at $x = 0$ and $x = r > 1$
$y = 0$ at $x = 0$ and $y = r^r e^{1-r}$ at $x = r$.

There are only two critical points so that $(0,0)$ is a local minimum and $(r, r^r e^{1-r})$ is a maximum.

Moreover, because the curve passes through $(1,1)$, we must have $r^r e^{1-r} > 1$.

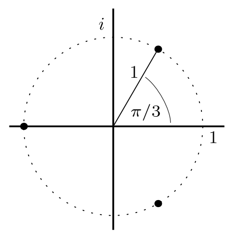
For $r = 2$ the maximum is at $(2, 4/e)$.

9. (a)



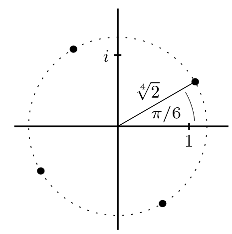
$z^2 = i = e^{i\pi/2}$.
So one root is $z = e^{i\pi/4}$.
Other roots increment by $e^{i2\pi/2} = e^{i\pi}$, so that
$z = e^{i\pi/4 + ik\pi}$
for $k = 0, 1$.

(b)



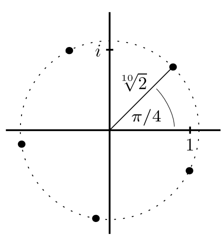
$z^3 = -1 = e^{i\pi}$.
So one root is $z = e^{i\pi/3}$.
Other roots increment by $e^{i2\pi/3}$, so that
$z = e^{i\pi/3 + ik2\pi/3}$
for $k = 0, 1, 2$.

⋆(c)



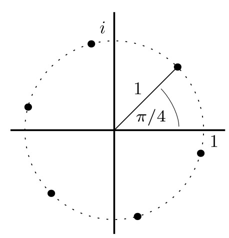
$z^4 = -1 + i\sqrt{3} = 2\,e^{i2\pi/3}$.
So one root is $z = \sqrt[4]{2}\,e^{i\pi/6}$.
Other roots increment by $e^{i2\pi/4} = e^{i\pi/2}$, so that
$z = \sqrt[4]{2}\,e^{i\pi/6 + ik\pi/2}$
for $k = 0, 1, 2, 3$.

(d)



$z^5 = -1 - i = \sqrt{2}\,e^{i5\pi/4}$.
So one root is $z = \sqrt[10]{2}\,e^{i\pi/4}$.
Other roots increment by $e^{i2\pi/5}$, so that
$z = \sqrt[10]{2}\,e^{i\pi/4 + ik2\pi/5}$
for $k = 0, 1, 2, 3, 4$.

(e)



$z^6 = -i = e^{i3\pi/2}$.
So one root is $z = e^{i\pi/4}$.
Other roots increment by $e^{i2\pi/6} = e^{i\pi/3}$, so that
$z = e^{i\pi/4 + ik\pi/3}$
for $k = 0, 1, 2, 3, 4, 5$.

⋆10. (a)
$$\begin{aligned} e^{i4\theta} &= \cos(4\theta) + i\sin(4\theta) = (\cos\theta + i\sin\theta)^4 \\ &= \cos^4\theta + 4\cos^3\theta i\sin\theta + 6\cos^2\theta i^2\sin^2\theta + 4\cos\theta i^3\sin^3\theta + i^4\sin^4\theta \\ &= \cos^4\theta - 6\cos^2\theta\sin^2\theta + \sin^4\theta + i(4\cos^3\theta\sin\theta - 4\cos\theta\sin^3\theta) \end{aligned}$$

Equating real parts gives $\cos(4\theta) = \cos^4\theta - 6\cos^2\theta\sin^2\theta + \sin^4\theta$.

(b) Equating real parts gives $\sin(4\theta) = 4\cos^3\theta\sin\theta - 4\cos\theta\sin^3\theta$.

(c) Using $\cos x = \cosh(ix)$ and $i\sin x = \sinh(ix)$ gives

$$\begin{aligned} \cosh(i4\theta) &= \cosh^4(i\theta) - 6\cosh^2(i\theta)\frac{\sinh^2(i\theta)}{i^2} + \frac{\sinh^4(i\theta)}{i^4} \\ &= \cosh^4(i\theta) + 6\cosh^2(i\theta)\sinh^2(i\theta) + \sinh^4(i\theta) \end{aligned}$$

So, setting $i\theta = A$ gives: $\cosh(4A) = \cosh^4 A + 6\cosh^2 A\sinh^2 A + \sinh^4 A$.

Also: $\dfrac{\sinh(i4\theta)}{i} = 4\cosh^3(i\theta)\dfrac{\sinh(i\theta)}{i} - 4\cosh(i\theta)\dfrac{\sinh^3(i\theta)}{i^3}$

and so $\sinh(i4\theta) = 4\cosh^3(i\theta)\sinh(i\theta) + 4\cosh(i\theta)\sinh^3(i\theta)$

So, setting $i\theta = A$ gives: $\sinh(4A) = 4\cosh^3 A\sinh A + 4\cosh A\sinh^3 A$.

11. (a) $y = e^{x^2}\sqrt{1+x}\,\sin^2 x$: $\dfrac{\mathrm{d}y}{\mathrm{d}x} = 2xe^{x^2}\sqrt{1+x}\,\sin^2 x + e^{x^2}\dfrac{1/2}{\sqrt{1+x}}\sin^2 x + e^{x^2}\sqrt{1+x}\,2\sin x\cos x$

(b) $x^2 - 4xy + y^2 = \dfrac{2}{xy}$: $2x - 4y - 4x\dfrac{\mathrm{d}y}{\mathrm{d}x} + 2y\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{-2}{(xy)^2}\big(y + x\dfrac{\mathrm{d}y}{\mathrm{d}x}\big)$

$$\Big(\frac{2x}{(xy)^2} + 2y - 4x\Big)\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{-2y}{(xy)^2} - 2x + 4y$$

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{2y - x - 1/(x^2y)}{1/(xy^2) + y - 2x}$$

(c) $x = \cos\theta,\ y = \sin\theta$: $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{\mathrm{d}y/\mathrm{d}\theta}{\mathrm{d}x/\mathrm{d}\theta} = \dfrac{\cos\theta}{-\sin\theta} = -\cot\theta$

⋆(d) $y = t^3 - t,\ x = t^2 - t$: $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{\mathrm{d}y/\mathrm{d}t}{\mathrm{d}x/\mathrm{d}t} = \dfrac{3t^2-1}{2t-1}$

⋆(e) $y - e^{-xy} = x - e^{xy}$: $\dfrac{\mathrm{d}y}{\mathrm{d}x} - e^{-xy}\big(-y - x\dfrac{\mathrm{d}y}{\mathrm{d}x}\big) = 1 - e^{xy}\big(y + x\dfrac{\mathrm{d}y}{\mathrm{d}x}\big)$

$$\big(1 + xe^{-xy} + xe^{xy}\big)\frac{\mathrm{d}y}{\mathrm{d}x} = 1 - ye^{xy} - ye^{-xy}$$

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1 - y(e^{xy} + e^{-xy})}{1 + x(e^{xy} + e^{-xy})}$$

(f) $y = \dfrac{x^2e^{1-x}\sqrt{x^2+2}}{(x+3)(x^2-1)}$: $\ln y = 2\ln x + 1 - x + \tfrac{1}{2}\ln(x^2+2) - \ln(x+3) - \ln(x^2-1)$

$$\frac{1}{y}\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{2}{x} - 1 + \tfrac{1}{2}\frac{2x}{x^2+2} - \frac{1}{x+3} - \frac{2x}{x^2-1}$$

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{x^2e^{1-x}\sqrt{x^2+2}}{(x+3)(x^2-1)}\Big(\frac{2}{x} - 1 + \frac{x}{x^2+2} - \frac{1}{x+3} - \frac{2x}{x^2-1}\Big)$$

12. (a) True. $e^{i\pi} = \cos\pi + i\sin\pi = -1$

(b) True. $e^{i2\pi} = \cos(2\pi) + i\sin(2\pi) = 1$

(c) True. $e^{i\pi/3} + e^{-i\pi/3} = \cos(\pi/3) + i\sin(\pi/3) + \cos(\pi/3) - i\sin(\pi/3) = 2\cos(\pi/3) = 2\times\tfrac{1}{2} = 1$

(d) True. $e^{i2\pi/3} + e^{-i2\pi/3} = \cos(2\pi/3) + i\sin(2\pi/3) + \cos(2\pi/3) - i\sin(2\pi/3) = 2\cos(2\pi/3) = 2\times(-\tfrac{1}{2}) = -1$

(e) $az^2 + bz + c = 0$ has the roots $z = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ which would be complex if $b^2 - 4ac < 0$.
There are two roots unless $b^2 = 4ac$ when there is only one root.
The statement is true only if we interpret there being two identical roots whenever $b^2 = 4ac$.

(f) True. To have a non-real root we must have $b^2 - 4ac < 0$, in which case the other root is also not real.

⋆13. (a)

$$\frac{1}{1+x} = \frac{1}{1+x} \qquad \frac{1}{1+0} = 1$$

$$\frac{\mathrm{d}}{\mathrm{d}x}\frac{1}{1+x} = \frac{-1}{(1+x)^2} \qquad \frac{-1}{(1+0)^2} = -1$$

$$\frac{\mathrm{d}^2}{\mathrm{d}x^2}\frac{1}{1+x} = \frac{1\times 2}{(1+x)^3} \qquad \frac{2!}{(1+0)^3} = 2!$$

$$\frac{\mathrm{d}^3}{\mathrm{d}x^3}\frac{1}{1+x} = \frac{-3!}{(1+x)^4} \qquad \frac{-3!}{(1+0)^4} = -3!$$

giving

$$\frac{1}{1+x} = 1 - x + 2!\frac{x^2}{2!} - 3!\frac{x^3}{3!} + \cdots = 1 - x + x^2 - x^3 + \cdots = \sum_{k=0}^{\infty}(-1)^k x^k$$

(b) $\dfrac{\mathrm{d}}{\mathrm{d}z}\tan^{-1} z = \dfrac{1}{1+z^2} = \displaystyle\sum_{k=0}^{\infty}(-1)^k z^{2k}$ so $\tan^{-1} z = \displaystyle\int\sum_{k=0}^{\infty}(-1)^k z^{2k}\,\mathrm{d}z = C + \sum_{k=0}^{\infty}(-1)^k\frac{z^{2k+1}}{2k+1}$

At $z = 0$: $C = \tan^{-1} 0 = 0$, so that $\tan^{-1} z = \displaystyle\sum_{k=0}^{\infty}(-1)^k\frac{z^{2k+1}}{2k+1}$.

(c) The radius of convergence should be the same as the series for $\dfrac{1}{1+z^2}$ which has singularities at $z = \pm i$.
Thus the distance to the nearest singularity is $|\pm i - 0| = 1$, making the radius of convergence 1.

14. (a) Polar coordinates (r,θ) are defined parametrically so that $x = r\cos\theta$ and $y = r\sin\theta$

i. With θ constant: $\left.\frac{dx}{dr}\right|_\theta = \cos\theta$ and $\left.\frac{dy}{dr}\right|_\theta = \sin\theta$, so $\left.\frac{dy}{dx}\right|_\theta = \left.\frac{dy}{dr}\right|_\theta \Big/ \left.\frac{dx}{dr}\right|_\theta = \frac{\sin\theta}{\cos\theta} = \tan\theta$

ii. With r constant: $\left.\frac{dx}{dr}\right|_r = -r\sin\theta$ and $\left.\frac{dy}{dr}\right|_r = r\cos\theta$, so $\left.\frac{dy}{dx}\right|_r = \left.\frac{dy}{dr}\right|_r \Big/ \left.\frac{dx}{dr}\right|_r = \frac{r\cos\theta}{-r\sin\theta} = -\cot\theta$

iii. Where the curves intersect (having the same values of r and θ) we find that

$$\left.\frac{dy}{dx}\right|_\theta \times \left.\frac{dy}{dx}\right|_r = \tan\theta\cot\theta = -1$$

showing that the curves intersect at right angles.

(b) Parabolic coordinates (u,v) are defined parametrically so that $2x = u^2 - v^2$ and $y = uv$.

i. With v constant: $2\left.\frac{dx}{du}\right|_v = 2u$ and $\left.\frac{dy}{du}\right|_v = v$, so $\left.\frac{dy}{dx}\right|_v = \left.\frac{dy}{dr}\right|_v \Big/ \left.\frac{dx}{dr}\right|_v = v/u$

ii. With u constant: $2\left.\frac{dx}{dv}\right|_u = -2v$ and $\left.\frac{dy}{dv}\right|_u = u$, so $\left.\frac{dy}{dx}\right|_u = \left.\frac{dy}{dr}\right|_u \Big/ \left.\frac{dx}{dr}\right|_u = -u/v$

iii. Where the curves intersect (having the same values of u and v) we find that

$$\left.\frac{dy}{dx}\right|_v \times \left.\frac{dy}{dx}\right|_v = \frac{v}{u} \times \left(-\frac{u}{v}\right) = -1$$

showing that the curves intersect at right angles.

(c) Elliptic coordinates (s,t) are defined such that $x = \cosh s\cos t$ and $y = \sinh s\sin t$.

i. With s constant: $2\left.\frac{dx}{dt}\right|_s = -\cosh s\sin t$ and $\left.\frac{dy}{dt}\right|_s = \sinh s\cos t$

so $\left.\frac{dy}{dt}\right|_s = \left.\frac{dy}{dt}\right|_s \Big/ \left.\frac{dx}{dt}\right|_s = \frac{\sinh s\cos t}{-\cosh s\sin t} = -\tanh s\cot t$

ii. With t constant: $2\left.\frac{dx}{ds}\right|_t = \sinh s\cos t$ and $\left.\frac{dy}{ds}\right|_t = \cosh s\sin t$

so $\left.\frac{dy}{ds}\right|_t = \left.\frac{dy}{ds}\right|_t \Big/ \left.\frac{dx}{ds}\right|_t = \frac{\cosh s\sin t}{\sinh s\cos t} = \coth s\tan t$

iii. Where the curves intersect (having the same values of s and t) we find that

$$\left.\frac{dy}{dx}\right|_s \times \left.\frac{dt}{dx}\right|_v = -\tanh s\cot t \times \coth s\tan t = -1$$

showing that the curves intersect at right angles.

iv. Since $x^2 = \cosh^2 s\cos^2 t$ and $y^2 = \sinh^2 s\sin^2 t$:

- with s constant, the relation $\cos^2 t + \sin^2 t = 1$ shows that $\frac{x^2}{\cosh^2 s} + \frac{y^2}{\sinh^2 s} = 1$ which represents ellipses that intersect the x-axis at $x = \pm\cosh s$, and the y-axis at $y = \pm\sinh s$.
- with t constant, the relation $\cosh^2 s - \sinh^2 s = 1$ shows that $\frac{x^2}{\cos^2 t} - \frac{y^2}{\sin^2 t} = 1$ which represents hyperbolae that intersect the x-axis at $x = \pm\cos t$, having asymptotes $y = \pm x\tan t$ for large values of $|x|$ and $|y|$.