

c: More Examples (by parts)

14'

Example. $\int \sqrt{1+x^2} dx$

Exercise 1. $\int \sqrt{1-x^2} dx$

Exercise 2. $\int \sqrt{x^2-1} dx$

d: Substitution in Definite Integrals^{22'}

Using substitution, definite integrals can be calculated in two ways.

either

1. The end-points must take the values of the new variable. That is:

$$\int_a^b f(x) dx = \int_{u(a)}^{u(b)} f(x(u)) \frac{dx}{du} du$$

or

2. Use substitution to find the indefinite integral. Then apply the end points. That is:

$$\text{if } \int f(x) dx = \int f(x(u)) \frac{dx}{du} du = F(x) + C$$

$$\text{then } \int_a^b f(x) dx = [F(x)]_a^b$$

Example. $\int_{1/2}^2 xe^{-x^2} dx$

e: Integrals of Quotients

22''

Rearranging a quotient often helps:

Example 1. $\int \frac{2x^2 - 3x + 2}{4x^2 - 4x + 5} dx$

Example 2. $\int \frac{2x^2 - 3x + 2}{\sqrt{4x^2 - 4x + 5}} dx$

e: Partial Fractions (repeated factor)²⁵

Example. $\frac{4x^2 - 15x + 12}{(1-x)(2x-3)^3}$