c: More Examples (by parts)

Example. $\displaystyle\int \sqrt{1+x^2}\,\mathrm{d}x$

Exercise 1. $\displaystyle\int \sqrt{1-x^2}\,\mathrm{d}x$

Exercise 2. $\displaystyle\int \sqrt{x^2-1}\,\mathrm{d}x$

d: Substitution in Definite Integrals

Using substitution, definite integrals can be calculated in two ways.

either

1. The end-points must take the values of the new variable. That is:

$$\int_a^b f(x)\,\mathrm{d}x = \int_{u(a)}^{u(b)} f\big(x(u)\big)\frac{\mathrm{d}x}{\mathrm{d}u}\,\mathrm{d}u$$

or

2. Use substitution to find the indefinite integral. Then apply the end points. That is:

if $\displaystyle\int f(x)\,\mathrm{d}x = \int f\big(x(u)\big)\frac{\mathrm{d}x}{\mathrm{d}u}\,\mathrm{d}u = F(x) + C$

then $\displaystyle\int_a^b f(x)\,\mathrm{d}x = \big[F(x)\big]_a^b$

Example. $\displaystyle\int_{1/2}^{2} xe^{-x^2}\,\mathrm{d}x$

e: Integrals of Quotients

Rearranging a quotient often helps:

Example 1. $\displaystyle\int \frac{2x^2 - 3x + 2}{4x^2 - 4x + 5}\,\mathrm{d}x$

Example 2. $\displaystyle\int \frac{2x^2 - 3x + 2}{\sqrt{4x^2 - 4x + 5}}\,\mathrm{d}x$

e: Partial Fractions (repeated factor)

Example. $\displaystyle\frac{4x^2 - 15x + 12}{(1 - x)(2x - 3)^3}$