Cartesian and Polar Coordinates

The precise location of a point in a plane can be identified using 2 axes at right angles ($\frac{\pi}{2}$ or 90°) intersecting at an "origin"

- If the distance of the point from the axes is
  - x in the direction of the most clockwise axis
  - y in the direction of the other axis

  then the point is at (x, y) in Cartesian coordinates
- If the line from the origin to the point:
  - has length r
  - is rotated an angle θ from the most clockwise axis

  then the point is at (r, θ) in polar coordinates

Note: conventionally, angles are measured anti-clockwise

From simple geometry

$$r = \sqrt{x^2 + y^2}$$
$$x = r\cos(\theta)$$
$$y = r\sin(\theta)$$

Graphs and Curve-Sketching

Using axes, algebraic relationships between two real variables can be drawn as a 'graph' in Cartesian form

Example 1. Sketch the graph of $\dfrac{x}{5} + \dfrac{y}{2} = 1.$

Example 2. Sketch $s = 3$ for any t in the (t, s) plane. Also sketch $t = 5$.

Example 3. Sketch $xy = b^2$.

What is represented in the case $b = 0$?

How would you sketch $xy = -b^2$?

Sketching Parametric Relationships

Sometimes two variables are each given in terms of a third, tracing out a curve as the third variable changes

Example 1. Sketch (u, v) with $u = t^3 - t, \; v = t^2$ for $t \in \mathbb{R}$

Example 2. Sketch (x, y) for $\theta \in \mathbb{R}$ with $x = 2\cos(\theta), \; y = \sin(2\theta)$

Example 3. Sketch (r, θ), in polar form, with $r = \sqrt{s}, \; \theta = \pi s$ for $s > 0$

generic equations for 'Conic Sections'

***parabola*:** $\boxed{x^2 = 4cy}$

'focus' is at $(0, c)$

***ellipse*:** $\boxed{\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1}$

foci are at $(\pm c, 0)$
for $c^2 = a^2 - b^2$
if $a^2 \geq b^2$

What is the curve if $a = b$?
Where are the foci if $b^2 > a^2$?

***hyperbola*:** $\boxed{\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1}$

asymptotes $y = \pm \frac{b}{a} x$
foci are at $(\pm c, 0)$
for $c^2 = a^2 + b^2$

Sketch the curves, asymptotes and foci of $\frac{y^2}{b^2} - \frac{x^2}{a^2} = 1$.