Relevant aspects of set-theory

There is much that can be said about sets but, for the moment, it is only necessary to know that a 'set' is a collection of items, called 'members' or 'elements' of the set. The precise definition of a set amounts to having a precise definition of all of its members.

The following notation and definitions are used in this text:

- A pair of curly brackets { } can be used to represent a set, with all items listed between the curly brackets being members of the set.
- The symbol $\in$, as used in '$\ell \in \mathbb{N}$,' means 'is a member of the set' and $\notin$ means 'is not a member of the set'. Thus

  $$63064 \in \mathbb{N} \text{ means '63064 is a member of the set } \mathbb{N}\text{'}$$

  and

  $$0 \notin \mathbb{N} \text{ means '0 is not a member of the set } \mathbb{N}\text{'.}$$

  That is why '$m \in \mathbb{N}$' is actually a mathematical statement (in fact a full sentence).
- Another statement, that could be used to define fully the set of natural numbers $\mathbb{N}$, is

  $$\mathbb{N} = \left\{ n \,\middle|\, n \geq 1 \text{ and either } n = 1 \text{ or } n - 1 \in \mathbb{N} \right\}.$$

  A translation into English words is: $\mathbb{N}$ is equal to the set consisting of all elements, such as n, which are such that n is greater than or equal to one and either n is one or n minus one is in the set $\mathbb{N}$.
- The vertical line | used in this definition can be thought of as meaning 'such that'.
- In any definition of a set, such as $\{-2, 9, -2, 0, 3, 0\}$, repeated items only count once. Either something is or it isn't a member; there is no such thing as repeated membership. The order is also irrelevant because there is no such thing as being the first, second or third member, and so on. Thus, we can write

  $$\{-2, 9, -2, 0, 3, 0\} = \{-2, 9, 0, 3\} = \{-2, 0, 3, 9\} = \{9, 3\ -2, 0, \}$$

  with all possible orderings defining exactly the same set.
- Any set that has only a finite number of elements is called a finite set. A finite set is always 'countable'. Quite literally, the members of the set can be counted.
- If all of the members of a set A are also in a set B, which might of course have more members, then A is said to be a 'subset' of B and B is called a 'superset' of A. These statements are written symbolically using the symbols $\subset$ and $\supset$, respectively:

  $$\text{That is} \quad \mathrm{A} \subset \mathrm{B} \quad \text{or} \quad \mathrm{B} \supset \mathrm{A} \quad \text{means} \quad \text{'if } x \in \mathrm{A} \text{ then } x \in \mathrm{B}.\text{'}$$

- If the sets A and B are equal then $\mathrm{A} \subset \mathrm{B}$ and $\mathrm{B} \subset \mathrm{A}$.
- For any set A it is always true that $A \subset A$.
- The union of two sets is another set that contains the combined membership of both sets. The union of the sets A and B is defined as

  $$\mathrm{A} \cup \mathrm{B} = \left\{ x \,\middle|\, x \in \mathrm{A} \text{ or } x \in \mathrm{B} \right\}.$$

- The intersection of two sets is a set that contains only the elements that are common to both sets. The intersection of the sets A and B is defined as

  $$\mathrm{A} \cap \mathrm{B} = \left\{ x \,\middle|\, x \in \mathrm{A} \text{ and } x \in \mathrm{B} \right\}.$$

- Finally, there is one set, called the empty set, that has no members at all; it is assigned the symbol $\emptyset$. If A is any set at all, then it is always true that $\emptyset \subset \mathrm{A}$.