

Some basic derivatives:

| $f(x)$ | $f'(x)$ | $f(x)$ | $f'(x)$ |
|---------------------------|------------------------------------|----------------------------|--------------------------------------|
| x^n | nx^{n-1} | e^x | e^x |
| $\ln(x)$ | $1/x$ | $\sin(x)$ | $\cos(x)$ |
| $\cos(x)$ | $-\sin(x)$ | $\tan(x)$ | $\sec^2(x)$ |
| $\cot(x)$ | $-\operatorname{cosec}^2(x)$ | $\sec(x)$ | $\sec(x) \tan(x)$ |
| $\operatorname{cosec}(x)$ | $-\operatorname{cosec}(x) \cot(x)$ | $\tan^{-1}(x)$ | $1/(1+x^2)$ |
| $\sin^{-1}(x)$ | $1/\sqrt{1-x^2}$ for $ x < 1$ | $\cos^{-1}(x)$ | $-1/\sqrt{1-x^2}$ for $ x < 1$ |
| $\sinh(x)$ | $\cosh(x)$ | $\cosh(x)$ | $\sinh(x)$ |
| $\tanh(x)$ | $\operatorname{sech}^2(x)$ | $\coth(x)$ | $-\operatorname{cosech}^2(x)$ |
| $\operatorname{sech}(x)$ | $-\operatorname{sech}(x) \tanh(x)$ | $\operatorname{cosech}(x)$ | $-\operatorname{cosech}(x) \coth(x)$ |
| $\sinh^{-1}(x)$ | $1/\sqrt{x^2+1}$ | $\cosh^{-1}(x)$ | $1/\sqrt{x^2-1}$ for $x > 1$ |
| $\tanh^{-1}(x)$ | $1/(1-x^2)$ for $ x < 1$ | $\coth^{-1}(x)$ | $-1/(x^2-1)$ for $ x > 1$ |

Basic rules for differentiation and integration:

- $\frac{d}{dx} (f(x) + g(x)) = \frac{d}{dx} f(x) + \frac{d}{dx} g(x) = f'(x) + g'(x)$ *derivative of a sum*
- $\frac{d}{dx} (cf(x)) = c \frac{d}{dx} f(x) = cf'(x)$ *derivative with a constant factor*
- $\frac{d}{dx} (f(x)g(x)) = f(x)g'(x) + f'(x)g(x)$ *derivative of a product*
“first times derivative of second plus second times derivative of first”
- $\frac{d}{dx} \frac{f(x)}{g(x)} = \frac{g(x)f'(x) - f(x)g'(x)}{g^2(x)}$ *derivative of a quotient*
“bottom times derivative of top minus top times derivative of bottom, over bottom squared”
- $\frac{d}{dx} f(g(x)) = f'(g(x))g'(x)$ *chain rule, or function of a function rule*
“derivative of the outer function times derivative of the inner function”
- $\frac{d}{dx} f(ax + b) = af'(ax + b)$ *special case of the chain rule*
- $\int f'(ax + b) dx = \frac{1}{a}f(ax + b) + C$ *integral of a function of a linear function*
- $\int f'(g(x))g'(x) dx = f(g(x)) + C$ *integral of a chain-rule derivative*
- $\int (f(x) + g(x)) dx = \int f(x) dx + \int g(x) dx$ *integral of a sum*
- $\int cf(x) dx = c \int f(x) dx$ *integral with a constant factor*
- $\int f(x)g'(x) dx = f(x)g(x) - \int f'(x)g(x) dx$ *integration by parts*
“one part times integral of other, minus integral of derivative of the one times integral of other”

Some basic integrals:

| $f(x)$ | $\int f(x) dx$ |
|---|---|
| e^x | $e^x + C$ |
| $x^n \text{ for } n \neq -1$ | $\frac{x^{n+1}}{n+1} + C$ |
| $1/x \text{ for } x \neq 0$ | $\ln x + C$ |
| $a^x \text{ or } e^{x \ln(a)} \text{ for } a \neq 1, a > 0$ | $\frac{a^x}{\ln(a)} + C$ |
| $e^{ax} \text{ for } a \neq 0$ | $\frac{e^{ax}}{a} + C$ |
| $\cos(ax) \text{ for } a \neq 0$ | $\frac{1}{a} \sin(ax) + C$ |
| $\sin(ax) \text{ for } a \neq 0$ | $-\frac{1}{a} \cos(ax) + C$ |
| $\frac{1}{x^2 + a^2} \text{ for } a \neq 0$ | $\frac{1}{a} \tan^{-1}(x/a) + C$ |
| $\frac{1}{a^2 - x^2} \text{ for } x < a , a \neq 0$ | $\frac{1}{a} \tanh^{-1}(x/a) + C$ |
| $\frac{1}{x^2 - a^2} \text{ for } x > a , a \neq 0$ | $-\frac{1}{a} \coth^{-1}(x/a) + C$ |
| $\frac{1}{x^2 - a^2} \text{ for } x \neq a , a \neq 0$ | $\frac{1}{2a} \ln \left \frac{x-a}{x+a} \right + C$ |
| $\frac{1}{\sqrt{x^2 + a^2}}$ for $a \neq 0$ | $\sinh^{-1}(x/a) + C$ |
| $\frac{1}{\sqrt{a^2 - x^2}}$ for $ x < a, a > 0$ | $\sin^{-1}(x/a) + C$ |
| $\frac{1}{\sqrt{x^2 - a^2}}$ for $x > a, a > 0$ | $\cosh^{-1}(x/a) + C$ |
| $\frac{1}{\sqrt{x^2 - a^2}}$ for $x < -a, a > 0$ | $-\cosh^{-1}(-x/a) + C$ |

definite integral: $\int_a^b f(x) dx = - \int_b^a f(x) dx$
 $\int_a^b f'(x) dx = [f(x)]_a^b = \lim_{x \rightarrow b^-} f(x) - \lim_{x \rightarrow a^+} f(x)$
provided $f'(x)$ is continuous for all $a < x < b$.

substitution: $\int f(x) dx = \int f(x(u)) \frac{dx}{du} du$ *indefinite integral*
 $\int_a^b f(x) dx = \int_{u(a)}^{u(b)} f(x(u)) \frac{dx}{du} du$ *definite integral*

integration by parts:

$$\int f(x)g'(x) dx = f(x)g(x) - \int f'(x)g(x) dx \quad \text{i}n\text{definite integral}$$

$$\int_a^b f(x)g'(x) dx = [f(x)g(x)]_a^b - \int_a^b f'(x)g(x) dx \quad \text{d}efinite integral$$